



Four Experiences and Some Reflections about the Influence of Mathematical Software on the Mathematics Curriculum

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JSRR/2015/17798

Editor(s):

(1) Ana Paula da Silveira Simões Pedro, Department of Education, University of Aveiro, Portugal.

Reviewers:

(1) Anonymous, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

(2) Anonymous, University of Mersin, Turkey.

Complete Peer review History: <http://www.sciencedomain.org/review-history.php?iid=1126&id=22&aid=9058>

Opinion Article

Received 26th March 2015

Accepted 14th April 2015

Published 30th April 2015

ABSTRACT

So far, the use of media in education has always been applied to achieve established goals, and the selection of these goals was slightly affected by the use of the chosen media. However, the availability of mathematical software, and, more precisely, computer algebra systems, can influence scientific methodology and, consequently, also mathematics teaching methodology. Therefore, the mathematics curriculum could, and presumably should, be reconsidered. An analysis of the possible changes at intermediate levels of the mathematics curriculum is shown in this article. Moreover, the viability of some of them is proved through their experimentation in the classroom. The chosen examples deal with: the survival function, surfaces and their projections, probability density functions, and probability calculus. These experiences are also summarized in this article. Our hypothesis is very concrete: viable curricular changes made possible by the use of computer algebra systems in a certain propitious environment do exist. We do not analyze other didactical issues such as which curricular changes could be made and which of them should be

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implemented; which is the influence of the teacher's knowledge of the media and his / her motivation, etc.

Keywords: Computer algebra systems; elevator principle; mathematics curriculum; probability calculus; probability density functions; surfaces; survival function; teaching mathematics with technology.

1. INTRODUCTION

Historically, media used in mathematical creation and education [1] have affected the creation and education of mathematics. For example, the ruler and compass originated new solving methods, but they also created new problems.

Computers and, in particular, mathematical software, are powerful media tools that help solve many problems, although they create new ones. In education, that probably provokes a possible and desirable reorganization of the curricular components, principally contents and their sequence.

After reflecting on these changes, we came up with a simple method of analyzing them. We have also implemented classroom experiences from some of the proposals that prove the viability of these changes [2-7]. This investigation in mathematical education is the Ph.D. thesis of the first author, one of which advisors is the second author. We have discovered similar ideas and opinions in [8], to whom we would like to thank for his most valuable comments.

1.1 Media and Curriculum

The real world is studied in the classroom using models, built using didactical media, on which the student interacts in order to achieve his goals ([9], p. 97). The need and appropriateness of a medium must be given by the curriculum, not by the medium itself. However, sometimes this curriculum requirement is precisely due to the existence of the appropriate media. For example, the requirement that students can prove that a function is a probability density function (i.e. that it is always positive and encircles a unit area) without knowing integral calculus is due to the incorporation of Computer Algebra Systems (CASs). According to [10,11] CASs can (and should) be used when the problem is laborious or the algorithm that solves a step of secondary importance is not known □this use of CASs is analyzed in [12].

1.2 Computers

Computers have shown to be effective in many aspects of knowledge, work, management and

leisure. To reject its use in school would fracture education from its social context.

Computers have specific characteristics that other educational media do not have:

- Its use is not exclusive of educational processes (the media used at school are usually exclusively related to school),
- They are available at home (most of the others can only be found at school),
- They are mainly for personal use,
- They are strongly motivating.

1.3 Use of Software in Mathematical Education

Although general purpose programs (word processors, for example) or the Internet, calculators and even video game consoles [13] can be used in the mathematics classroom, we are mainly interested in computer applications specific to mathematical education. These applications cover a wide spectrum. We could distinguish:

- Closed packages, where interaction is nonexistent or almost nonexistent, analogous to the textbook (like programmed learning packages),
- Packages whose goal is to obtain a numerical, graphical or contrasting result (as happens with the usual statistical packages),
- Packages that allow for analysis, experimentation, discovery and creative thinking (as Dynamic Geometry Systems, CASs or spreadsheets) –see, for instance, [14].

Nevertheless, the general characteristics of a package do not condition its use [15]. A programmed learning package can be used to discuss it (then its use is no longer as a programmed learning package) or the most powerful CAS can be used to sum two rational numbers (then the environment created allows neither investigation nor analysis).

We shall neither treat here the risks of the use of computers [16,17] nor its real use in the classroom [18,19].

We do not analyze other issues such as which curricular changes could be made and which of them should be implemented; which is the influence of the teacher’s knowledge of the media and his / her motivation [20] etc.

2. MATERIALS AND METHODS

2.1 Influences of Mathematical Software in the Mathematics Curricula: an Organization Proposal

Computer programs are a type of media that influence curricular components. They allow for curricular changes that have to be analyzed and experimented. The method for analyzing curricular changes that we propose begins with the curricular components whose modification is at the discretion of the teachers (not all are at their reach: for example, the general goals are fixed by the government). Therefore, an analysis of the changes of the contents [2] is made reflecting upon:

- Those that can be incorporated (for example, those related to the tool itself),
- Those that can be suppressed (for example, the use of statistical tables),
- Those that can change somehow, strictly referring to their contents (for instance, function plotting at elementary levels), as well as what is the sequence (for example, probability in continuous random variable can be taught before integral calculus if the integrals are computed by a computer used as a black-box).

In summary, looking at all possible curricular changes including the appearance, disappearance, and change in the curricular components results in an exhaustive organization that we successfully used in our research. This exhaustive organization is summarized in Table 1 [6].

Other author’s proposals about changes in the curriculum (for example Kutzler’s “scaffolding principle” [10,21], or the “white-box / black-box

principle” [22,23] can be incorporated naturally in our organization.

Mathematics curriculum is especially delicate, as it is related to real life and not all skills are always acquired [24].

2.2 The elevator principle

We have denoted “elevator principle” to the possibility that, we believe, the computer has:

- a) To “lower” contents traditionally allocated, because of its difficulty, in courses higher than that of the student (the student theoretically has the mathematical knowledge to develop the topic, but in practice the topic presents several difficulties: extension, advanced vocabulary, complex algorithms, difficulties of the graphic representations...),
- b) Or to “raise” the student to contents that have to be allocated in higher courses because mathematical knowledge that the student doesn’t have are needed.

2.3 Experimental Details: Four Classroom Experiences

We have implemented and we have evaluated the viability of these changes in the classroom. Our research took place in a region in the interior of Spain with a relatively high dependency on agriculture and cattle raising.

The students that participated in two of the experiences were 17 years old and studied the “non-compulsory secondary education”, what corresponds, in the Spanish system, to the penultimate year before entering university. In the Spanish system, 16 years old students have the choice to follow “non-compulsory secondary education” or to enter vocational studies.

In the other two experiences the students were fourth year university students from a librarians school.

Table 1. Possible organization of curricular changes

	Appearances	Changes	Disappearances
Contents	Concepts Procedures Attitudes Sequence		
Methodology			
Assessment			

In all cases, a CAS (*DERIVE*TM¹) was used and the research was conducted in the students' usual classroom and followed their usual schedule. The students freely grouped themselves in pairs.

2.3.1 An experience about a change of the methodology: the survival function

This experience was carried out with 17 years old students. Fundamentally, apart from the appearance of the contents corresponding to the tool, there was a change of the contents' sequence (because of the use of the computer) and a change in the methodology (because of the presentation of a mathematical model, made possible by the use of the computer) [30]. In fact, changes in the methodology, due to the use of the computer, occurred in all the experiences carried out. The changes are organized in Table 2. This modelling process allowed for the presentation of a wider spectrum of applications to the student, even allowing for the inclusion of topics generally reserved for higher courses and to real scientific problem solving.

The model is that of survival functions [31]. The simplest case is that where the "hazard function" $h(x)$, is constant, that is, $h(x) = \lambda$. Then the "survivor function" $S(t)$ can be expressed:

$$S(t) = e^{-\int_0^t h(x) dx} = e^{-\int_0^t \lambda dx} = Ce^{-\lambda t}$$

This model can be found in many places in science, and is traditionally studied in different places, but normally at a higher level than that of the students in this experience. It can be found in pharmacokinetics (drug absorption and elimination), physiology (alveoli' air renewal), ecology (species survival), chemistry (disintegration), documentation (scientific literature interest decline), etc.

The students worked without major problems for three hours on topics belonging to different branches of knowledge.

¹*DERIVE*TM was initially developed and commercialized by the Hawaiian company SoftWare House, Inc. and later by Texas Instruments. Unfortunately, *DERIVE*TM is no longer sold. Nevertheless, it was a CAS commonly used for educational purposes [25,26], and was intensively used in some countries like Austria [27]. Surprisingly, it is still used, especially in secondary education. Moreover, these experiences could be carried out with the vast majority of the CASs that include graphical capabilities. For instance, similar experiences were developed during 2007 by these and other authors using Maxima (in the frame of a research project of the Extremadura regional government about teaching high-school mathematics with the help of a CAS [28,29]).

The previous experience corresponds to the case a).

2.3.2 An experience about incorporating contents: surfaces and their projections

We have also implemented this didactic experience with 17 years old students (during two consecutive years). The experience can be framed in the line of curricular changes that we have denoted "contents appearance". The changes of this particular experience are summarized in Table 3.

*DERIVE*TM has been used in this experience to create an adequate frame for the development of graphic representation processes. These processes appear in different subjects and can not be treated without the help of a computer because of the complexity of the necessary mathematical media.

For instance, let us consider the following surface:

$$e^{\left(-\frac{z}{5}\right)} \cdot \left(\frac{\pi}{2} - \arctan y\right) \tag{1}$$

Which plot appears in Fig. 1 the mathematical software allows to cut it through a pencil of horizontal planes:

$$z = k, k = 1, 2, 3, 4, 5 \tag{2}$$

(Fig. 2) and also allows to represent the projections of the intersection curves on the plane $z = 0$ (Fig. 3).

Therefore, the function is represented in an algebraic language in (1) and (2) and also in a graphical language through a perspective 3D drawing and plane sections.

From the algebraic point of view, the graphical representations come from considering z either as a parameter or as a dependent variable:

- In the first case we obtain a uniparametric family of functions plotted in the plane (for positive integer values of the parameter),
- In the second one we obtain a surface.

For instance, $xy = k$ is usually interpreted as a family of hyperbolae in the plane, meanwhile $xy = z$ is usually interpreted as a surface. Moreover, CASs normally allow the simultaneous visualization of the different expressions (Fig. 4).

Table 2. Curricular changes found in the first experience

		Appearances	Changes	Disappearances
Contents	Concepts			
	Procedures	•		
	Attitudes			
	Sequence		•	
Methodology			•	
Assessment			•	

Table 3. Curricular changes found in the second experience

		Appearances	Changes	Disappearances
Contents	Concepts	•		
	Procedures	•		
	Attitudes	•		
	Sequence			
Methodology			•	
Assessment				

In the classroom, the students began representing some surfaces, whose expressions are simple, graphically. Later, they studied the equations of planes parallel to two coordinate axes and the section of horizontal planes with different surfaces, that were projected on plane $z = 0$. Finally, families of horizontal planes, their sections with different surfaces and the projection of the latter on plane $z = 0$ were studied.

2.3.3 An experience about changing traditional sequence: probability density functions

Essentially, a change in the sequence (apart from the necessary appearance of procedural contents of the tool and the methodological changes that its use implies) is experimented in this work. Students can “ascend” to the required level of the mathematical building using a CAS (used as a “black-box” [22,23]) as an “elevator”. Let us observe that this is not exactly the “scaffolding principle” [10,21], because the student has not necessarily studied the topic [4]. In the particular case of this experience, integral calculus’ algorithms that these students did not know are eliminated. These changes are represented in Table 4.

The experience was carried out with students of Documentación (Information Science degree) at the School of Librarians of a Spanish university. These students were required to take an introductory course to inferential statistics. The program began with the concept of probability and continued with discrete and continuous

distributions. However, the students hadn’t necessarily acquired sufficient mathematical calculus knowledge in previous years.

The students had a two hour introductory practice to the CAS *DERIVE*TM, although some participants needed some extra time. The students learned in these practices how to plot functions and families of functions and how to compute definite integrals. Once this content of the tool was developed, the following sessions were devoted to the acquisition of the following mathematical contents:

- Check if a function is a probability density function, using its definition,
- Study the properties of some of the most frequent probability density functions, preferably from their plots,
- Study the behaviour of the parameters that appear in the expressions of these functions through the representation of families of functions depending on these parameters (as Fig. 5 shows for Student’s t-distribution).

2.3.4 An experience with several simultaneous curricular changes: probability calculus

A fourth classroom experience was developed at the same level as the previous one, during another academic year. In this one, although the main goal was to suppress algorithmic content, there was a deeper change in the selection and sequence of contents. An algorithm implemented in *DERIVE*TM allows the suppression of usual

procedural contents of continuous random variable (in all cases where they are usually used) ([32], p. 47). Moreover, the need to use variable changes and approximations of some distributions by others disappears. The procedures devoted to calculate the value of a variable for a certain probability and other similar ones, like to calculate the interval $(c-r, c+r)$ such that the probability of the variable taking a value in it is p , given either c or r , can all be suppressed. All these problems can be solved by the same method, simplifying in a unique procedural model the different procedures.

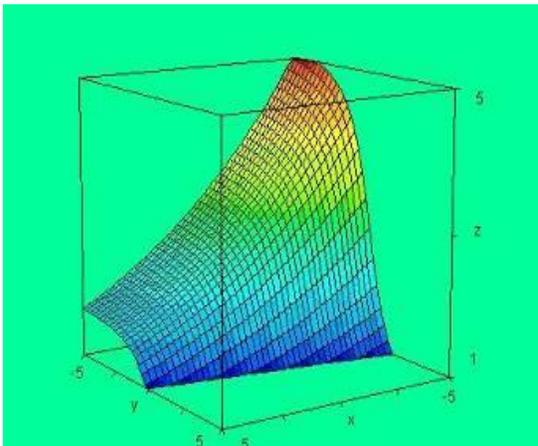


Fig. 1. Surface given by equation (1)

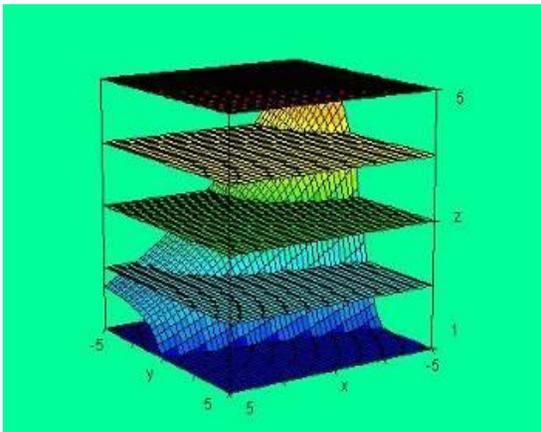


Fig. 2. Sections of surface (1) by the pencil of parallel planes (2)

On the other hand, probability calculus in continuous random variable can be taught in a formal way through the use of integral calculus, raising the students to a level they could not reach without the help of a CAS. These curricular changes are summarized in Table 5.

The contents began with a first block containing the contents of the experience described in the third experience.

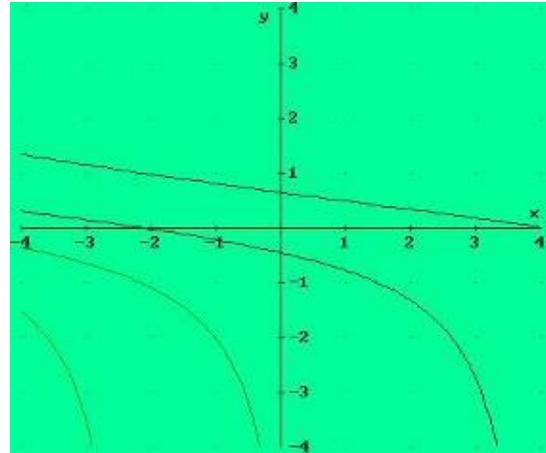


Fig. 3. Projection of the sections of surface (1) by the pencil of parallel planes (2) on plane $z=0$

A second block developed probability calculus in continuous random variable with two problems. The first one is: given an interval (a,b) , calculate the probability of the random variable being in this interval. The problem is solved using

$$P(a < X \leq b) = \int_a^b f(x)dx$$

that the CAS computes directly. The second problem is: given p and a (or b), calculate the interval $(a,b]$ such that

$$P(a < X \leq b) = p.$$

The corresponding equation is in this case

$$\int_a^b f(x)dx = p$$

where b (or a) is the unknown. This case can also be presented defining the interval in the form $(a, a+k)$ or $(c-r,c+r)$. The computer is able to solve these integral equations (in all forms) numerically.

In a third block of contents, the usual distributions (that is: the *normal distribution* $N(\mu,\sigma)$, *Pearson's χ^2 distribution*, *Student's t -distribution*, *Snedecor's F -distribution* and the *rectangular distribution*) were studied. We also looked at how changing the parameters influence each one. That the *Student's t -distribution* was an approximation of the *normal distribution* was

mentioned too, and the plots of both of them were compared. The average and variance of these distributions were also studied, as well as the probability calculations for the different ways of presenting an interval as mentioned above.

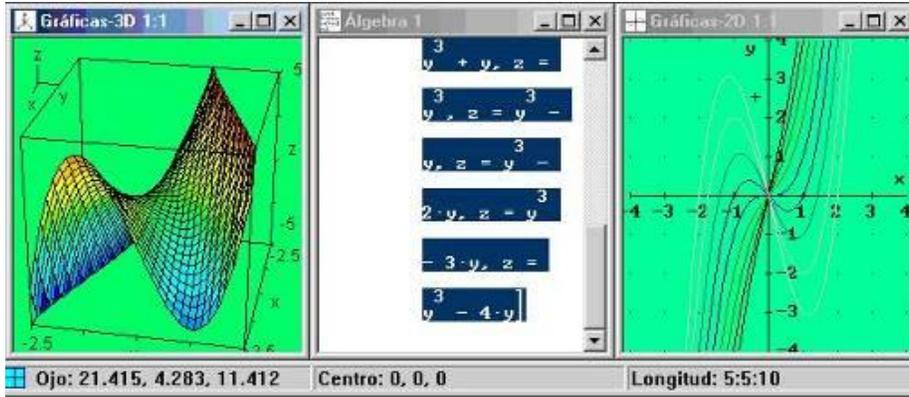


Fig. 4. Simultaneous presentation in *DERIVE*TM of a surface in different mathematical languages

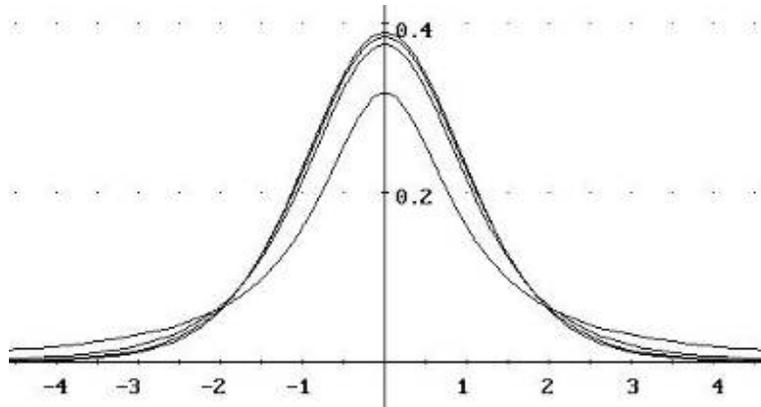


Fig. 5. Representation of student's t-distribution for different degrees of freedom

Table 4. Curricular changes found in the third experience

		Appearances	Changes	Disappearances
Contents	Concepts			
	Procedures	•		•
	Attitudes			
	Sequence		•	
Methodology		•		
Assessment				

Table 5. Curricular changes found in the fourth experience

		Appearances	Changes	Disappearances
Contents	Concepts			
	Procedures	•	•	•
	Attitudes			
	Sequence		•	
Methodology		•		
Assessment				

Finally an introduction to inferential statistics was presented. It was taken into account that, once a confidence level $1-\alpha$ is fixed, the radius of the interval centered in the sample average that covers the population average with this probability, can be found with the equation in r

$$\int_{\bar{x}-r}^{\bar{x}+r} \frac{1}{\frac{\sigma}{\sqrt{n}} \sqrt{2\pi}} e^{-0.5 \frac{(x-\bar{x})^2}{\sigma^2/n}} dx = 1-\alpha$$

that *DERIVE*[™] solves directly (numerically). The case when the variance is unknown is treated similarly, being unnecessary to treat large samples approximately.

The classroom experience was assessed through a contents test (that was not implemented specifically for the experience, but was integrated in the final exam instead), the teacher's observations, classroom observations performed by other two teachers of the Librarians School and interviews of the students.

3. RESULTS AND DISCUSSION

3.1 Results: First Experience

To assess contents acquisition they were given a text from a physiology textbook [33], discussing the elimination of an inert gas from the lungs. This was explained without mathematical notation.

The students were asked to find the parameters defining the function, to plot it and to solve a short algebraic problem. They were also asked to interpret a page containing a problem about electricity, related to the model. Both exercises were obtained from well-known university textbooks.

The assessment gave different results depending on the question. Most students answered some questions correctly meanwhile other questions were only answered by the best students. The correlation with other mathematics exams can be described as normal (0.65).

The students found other aspects of the experience very valuable: the media, the working environment and the motivation.

Therefore, we believe we have proved that it is possible to present contents usually allocated at higher levels in the scientific curriculum through modelling.

3.2 Results: Second Experience

The result of the experience was positive and we consider that the possibility of including new content in the present mathematics curriculum, that cannot be introduced without the use a computer (at those levels), has been proven.

The experience was very well rated (by students and teachers) regarding both media and design. It was rated acceptable regarding concept acquisition. The motivation of the students toward the work was high and could be concluded by the positive way in which they worked.

3.3 Results: Third Experience

The students worked with great interest with *DERIVE*[™]. Major difficulties neither arose in the session regarding the tool nor in those about mathematical content. The students were not given any more specific content about probability density functions after the experience. After a month had passed, they were given a pop quiz where a reasonable ratio of successes was obtained.

The vast majority of the students rated the experience positively. They easily got close to algebraic and graphic expressions, working concepts that they would not have had time to acquire otherwise. Our opinion about the design, development and results is very positive. So we believe we have proven how the difficulty to study a certain part of mathematics, generated by the lack of knowledge of the algorithms necessary for its development, can be bypassed substituting traditional procedures by those provided by technological media.

3.4 Results: Fourth Experience

Some interesting results are that the ratio of students that think that the work they were doing was not exactly "mathematical" is higher than the one obtained when assessing the science high-school groups (first and second experiences). Nevertheless, 87% of the students thought that it is better to use the computer. Furthermore, a high percentage (73%) mentioned that they prefer it to traditional methodology. Many students were concerned about assessment using computers because they have to add the contents of the tool to the mathematical contents. The graphical aspects are positively valued when acquiring concepts.

The experience's global development proves the viability of our proposal in the environment where it has been implemented. Therefore, we believe that the goal of showing an example of how the difficulty of studying a part of mathematics that comes from not knowing the necessary algorithms can be by-passed through the substitution of traditional procedures by those added by technological media, is reached. For instance, using mathematical tables can be avoided by using mathematical software. Moreover, CASs allow to simplify the number of procedures used, therefore approaching the students to the concepts that originated these procedures [5].

3.5 Discussion and Summary

To divide the curriculum into curricular components allows for a better and more detailed analysis of its possible modifications. This analysis can be applied to the specific case of modifications generated by mathematical software in mathematical education.

Using computer programs to "do mathematics" and, more precisely, using the CAS, allows for deeper and wider changes than simply algorithm reduction. We have proved as example that they create at least two possibilities in this sense:

- The first one, modelling natural and social phenomena, allows us to globalize its study, mathematizing different situations that can be found at different levels of education,
- The second one, modifying traditional calculus strategies into others that allow changing contents' sequence, raising the students into higher levels. We have proved that some elementary ideas about integral calculus allow developing a big portion of elementary statistics without having the risk of the student getting lost because of having to classify the kind of problem or to recognize the necessary distribution or table to use. Consequently, concepts are preserved more easily.

Curricular content also changes when computers and mathematical software are available: there are topics in mathematical education that can be introduced in the curriculum thanks to the use of mathematical software. This introduction is desirable; contents appearing, disappearing or been changed. Anyway, the interest of these changes has to be reviewed and experimented in each situation, according to its profitability.

Innovation in this direction can be easily implemented in each teacher's own curriculum, but shows important difficulties for its generalization, coming from social and educative states. The most important difficulty is its rejection in summative assessment processes, both inside and outside standardized education.

4. CONCLUSION

Mathematical software in general, and particularly CASs, allow for changes in traditional mathematics curriculum at intermediate educational levels. These curricular changes can be classified according to the curricular components. Their experimentation gave positive results: it encouraged motivation, classroom relations and concept acquisition and it improved the profitability of the time dedicated to learning. We do not analyze other didactical issues such as which curricular changes could be made and which of them should be implemented; which is the influence of the teacher's knowledge of the media and his/her motivation etc.

ACKNOWLEDGEMENTS

This work was partially supported by the research project *Modificaciones curriculares en matemáticas posibilitadas por las tecnologías de la información (ICE-Universidad de Extremadura)* and a sabbatical year granted to the first author by the Spanish *Ministerio de Educación y Ciencia*.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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