



Derived Reduced Balanced Incomplete Block Design

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Authors' contributions

This work was carried out in collaboration among all authors. All Authors assisted in the derivation of the concepts in the paper. The first author wrote and compiled the document while authors OF, KA and NE assisted in the proof reading and correction of the document to match the derived concepts. All authors read and approved the final manuscript.

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Method Article

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Abstract

Construction of Balanced Incomplete Block Designs (BIBD) is a combination problem that involves the arrangement of v treatments into b blocks each of size k such that each treatment is replicated exactly r times in the design and a pair of treatments occur together in λ blocks. Several methods of constructing BIBDs exist. However, these methods still cannot be used to design all BIBDs. Therefore, several BIBDs are still unknown because a definite construction method for all BIBDs is still unknown. The study aimed to develop a new construction method that could aid in constructing more BIBDs. The study derived a new class of BIBD from un-reduced BIBD with parameters v and k such that $k \geq 3$ through selection of all blocks within the un-reduced BIBD that contains a particular treatment i then in the selected blocks treatment delete treatment i and retain all the other treatments. The resulting BIBD was Derived Reduced BIBD with

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parameters $v^* = v - 1, b^* = \binom{v-1}{k-1}, k^* = k - 1, r^* = \binom{v-2}{k-2}, \lambda = \binom{v-3}{k-3}$. In conclusion, the construction method was simple and could be used to construct several BIBDs, which could assist in solving the problem of BIBD, whose existence is still unknown.

Keywords: Un-reduced BIBD; block; treatment; reduced; derived; incomplete block.

1 Introduction

Yate [1] developed the Balanced Incomplete Block Design (BIBD) to find solutions to treatment arrangements in the agricultural field with constrained block sizes. However, with time, the construction of BIBD has been determined to be a combinatorial problem that entails the arrangements of v treatments into b blocks of size k each such that each treatment occurs r times in the entire design and each pair of treatments occurs together λ number of times [2-11].

Over the years, much research in combinatorial mathematics has emphasized the construction of BIBDs to find solutions to a given set of BIBDs. The investigations have led to the development of different techniques for constructing BIBD. Most of these methods were first introduced by Bose [3], but later, other researchers also developed further methods of designing BIBDs [12-18]. The developments have led to the existing known methods of constructing BIBD, which include Projection Geometry, Euclidean Geometry, Cyclic Difference Sets Method, Symmetric Repeated Difference Method, Latin Square Method, Linear Integer Programming, Use of the IBD package in R software, and Using Existing BIBD Designs [19, 20, 21,5, 22,23, 24, 8,25].

Different classes of BIBD have been discovered through the construction of BIBD using other existing BIBDs. Some of these classes of BIBD include Derived, Residual, Dual, and Complementary BIBD [26, 27]. For a BIBD with parameters (v, b, r, k, λ) when a block from the BIBD is deleted and then in the remaining $b - 1$ blocks all the other treatments are deleted except the ones that were in the deleted block then a Derived Balanced Incomplete Block Design with parameters $v^* = v, b^* = b - 1, r^* = r - 1, k^* = k, \lambda^* = \lambda - 1$ is constructed from [26,27].

The un-reduced BIBD is considered a universal set of BIBDs with parameters v and k . The design comprises a set of all possible combinations $\binom{v}{k}$, each possible combination being a block. The design contains all other BIBDs with parameters v and k . This BIBD has parameters $v^* = v, b^* = \binom{v}{k}, r^* = \binom{v-1}{k-1}, k^* = k, \lambda^* = \binom{v-2}{k-2}$ [26,27].

The general overview of the literature shows that various scholars have done extensive work over the years on the construction of BIBDs. However, a severe gap exists because all these techniques can only construct some BIBDs. According to Jane [28], several BIBD designs still need to be determined whether to exist because despite them satisfying all the design parameters conditions, the construction methods cannot be used in their construction. The fact has left several BIBD designs still being discovered [29,30,5,28,22,10].

The present study intended to bridge this gap by introducing a new class of BIBD known as Derived Reduced Balanced Incomplete Block Design (DRBIBD), which uses the un-reduced BIBD to construct a new type of BIBD. The idea behind the technique was motivated by the fact that un-reduced BIBD was a universal set of several classes of BIBD. Therefore, all the unknown types of BIBDs can still be derived from the design using the appropriate techniques.

2 Materials and Methods

2.1 Construction of Derived Reduced Bibd

Theorem 1: Consider an un-reduced BIBD with parameters v and $k \geq 3$. When blocks from the design containing treatment i are extracted from the design and in the extracted blocks, treatment i is deleted from all

the blocks, leaving the rest of the treatments. Then a BIBD is formed with parameters $v^* = v - 1, b^* = \binom{v-1}{k-1}, k^* = k - 1, r^* = \binom{v-2}{k-2}, \lambda = \binom{v-3}{k-3}$.

Proof: We need to prove that the Derived Reduced BIBD satisfies the necessary conditions for the existence of a BIBD. First, we begin with the first condition.

$$\begin{aligned}
 b^*k^* &= \left(\frac{(v-1)!}{(k-1)!(v-k)!} \right) k - 1 \\
 &= \left(\frac{(v-1)!(k-1)}{(k-1)(k-2)!(v-k)!} \right) \\
 &= \frac{(v-1)!}{(k-2)!(v-k)!} \\
 &= \left(\frac{(v-1)(v-2)!}{(k-2)!(v-k)!} \right) \\
 &= \left(\frac{(v-2)!}{(k-2)!(v-k)!} \right) v - 1 \\
 &= r^*v^*
 \end{aligned}$$

Hence, the first condition of BIBD is satisfied. Next, we check the second condition for the existence of BIBD.

$$\begin{aligned}
 \lambda^*(v^* - 1) &= \left(\frac{(v-3)!}{(k-3)!(v-k)!} \right) (v-2) \\
 &= \left(\frac{(v-2)(v-3)!}{(k-3)!(v-k)!} \right) \\
 &= \frac{(v-2)!}{(k-3)!(v-k)!} \\
 &= \left(\frac{(k-2)(v-2)!}{(k-2)(k-3)!(v-k)!} \right) \\
 &= \left(\frac{(v-2)!}{(k-2)!(v-k)!} \right) (k-2) \\
 &= r^*(k^* - 1)
 \end{aligned}$$

The second condition of BIBD is also satisfied; therefore, the Derived Reduced BIBD is indeed a BIBD.

3 Results and Discussion

The study illustrated how the DRBIBD is from an un-reduced BIBD with parameters v and k . The illustration was as discussed below;

3.1 Illustration 1

Let's consider an un-reduced BIBD generated $v = 8$ treatments and $k = 7$ plots as shown in Table 1.

Table 1. Un-reduced BIBD with $v = 8$ and $k = 7$

Un-reduced BIBD Blocks
Block 1 = {1, 2, 3, 4, 5, 6, 7}
Block 2 = {1, 2, 3, 4, 5, 6, 8}
Block 3 = {1, 2, 3, 4, 5, 7, 8}
Block 4 = {1, 2, 3, 4, 6, 7, 8}
Block 5 = {1, 2, 3, 5, 6, 7, 8}
Block 6 = {1, 2, 4, 5, 6, 7, 8}
Block 7 = {1, 3, 4, 5, 6, 7, 8}
Block 8 = {2, 3, 4, 5, 6, 7, 8}

From the BIBD in Table 1, if we select only the blocks that contain treatment 1 from the design and from the chosen blocks delete treatment 1, then the BIBD illustrated in Table 2 will result.

Table 2. Derived Reduced BIBD with $v = 7, k = 6, b = 7, r = 6, \lambda = 5$

Resulting Derived Reduced BIBD Blocks
Block 1 = {2, 3, 4, 5, 6, 7}
Block 2 = {2, 3, 4, 5, 6, 8}
Block 3 = {2, 3, 4, 5, 7, 8}
Block 4 = {2, 3, 4, 6, 7, 8}
Block 5 = {2, 3, 5, 6, 7, 8}
Block 6 = {2, 4, 5, 6, 7, 8}
Block 7 = {3, 4, 5, 6, 7, 8}

3.2 Illustration 2

Let's consider an un-reduced BIBD generated $v = 6$ treatments and $k = 3$ plots as shown in Table 3.

Table 3. Un-reduced BIBD with $v = 6$ and $k = 3$

Un-reduced BIBD Blocks	
Block 1 = {1, 2, 3}	Block 11 = {2, 3, 4}
Block 2 = {1, 2, 4}	Block 12 = {2, 3, 5}
Block 3 = {1, 2, 5}	Block 13 = {2, 3, 6}
Block 4 = {1, 2, 6}	Block 14 = {2, 4, 5}
Block 5 = {1, 3, 4}	Block 15 = {2, 4, 6}
Block 6 = {1, 3, 5}	Block 16 = {2, 5, 6}
Block 7 = {1, 3, 6}	Block 17 = {3, 4, 5}
Block 8 = {1, 4, 5}	Block 18 = {3, 4, 6}
Block 9 = {1, 4, 6}	Block 19 = {3, 5, 6}
Block 10 = {1, 5, 6}	Block 20 = {4, 5, 6}

From the BIBD in Table 3, if we select only the blocks that contain treatment 6 from the design and from the chosen blocks delete treatments 6, then the BIBD illustrated in Table 4 will result.

Table 4. Derived Reduced BIBD with $v = 5, k = 2, b = 10, r = 4, \lambda = 1$

Resulting Derived Reduced BIBD Blocks	
Block 1 = {1, 2}	Block 6 = {2, 3}
Block 2 = {1, 3}	Block 7 = {2, 4}
Block 3 = {1, 4}	Block 8 = {2, 5}
Block 4 = {1, 5}	Block 9 = {3, 4}
Block 5 = {3, 5}	Block 10 = {4, 5}

3.3 List of Some Potential Derived Reduced BIBDs

The Derived Reduced BIBD technique can construct several BIBDs, as illustrated in Table 5.

Table 5. List of some Derived Reduced BIBD

No.	Un-reduced BIBD	Derived Reduced BIBD
1	$v = 4, k = 3$	$v = 3, b = 3, r = 2, k = 2, \lambda = 1$
2	$v = 5, k = 3$	$v = 4, b = 6, r = 3, k = 2, \lambda = 1$
3	$v = 5, k = 4$	$v = 4, b = 4, r = 3, k = 3, \lambda = 2$
4	$v = 6, k = 3$	$v = 5, b = 10, r = 4, k = 2, \lambda = 1$
4	$v = 6, k = 4$	$v = 5, b = 10, r = 6, k = 3, \lambda = 3$
5	$v = 6, k = 5$	$v = 5, b = 5, r = 4, k = 4, \lambda = 3$
6	$v = 7, k = 3$	$v = 6, b = 15, r = 5, k = 2, \lambda = 1$
7	$v = 7, k = 4$	$v = 6, b = 20, r = 10, k = 3, \lambda = 4$
8	$v = 7, k = 5$	$v = 6, b = 15, r = 10, k = 4, \lambda = 6$
9	$v = 7, k = 6$	$v = 6, b = 6, r = 5, k = 5, \lambda = 4$
10	$v = 8, k = 3$	$v = 7, b = 21, r = 6, k = 2, \lambda = 1$
11	$v = 8, k = 4$	$v = 7, b = 35, r = 15, k = 3, \lambda = 5$
12	$v = 8, k = 5$	$v = 7, b = 35, r = 20, k = 4, \lambda = 10$
13	$v = 8, k = 6$	$v = 7, b = 21, r = 15, k = 5, \lambda = 10$
14	$v = 8, k = 7$	$v = 7, b = 7, r = 6, k = 6, \lambda = 5$
15	$v = 9, k = 3$	$v = 8, b = 28, r = 7, k = 2, \lambda = 1$
16	$v = 9, k = 4$	$v = 8, b = 56, r = 21, k = 3, \lambda = 6$
17	$v = 9, k = 5$	$v = 8, b = 70, r = 35, k = 4, \lambda = 15$
18	$v = 9, k = 6$	$v = 8, b = 56, r = 35, k = 5, \lambda = 20$
19	$v = 9, k = 7$	$v = 8, b = 28, r = 21, k = 6, \lambda = 15$
20	$v = 9, k = 8$	$v = 8, b = 8, r = 7, k = 7, \lambda = 6$
21	$v = 10, k = 3$	$v = 9, b = 36, r = 8, k = 2, \lambda = 1$
22	$v = 10, k = 4$	$v = 9, b = 84, r = 28, k = 3, \lambda = 7$
23	$v = 10, k = 5$	$v = 9, b = 126, r = 56, k = 4, \lambda = 21$
24	$v = 10, k = 6$	$v = 9, b = 126, r = 70, k = 5, \lambda = 35$
25	$v = 10, k = 7$	$v = 9, b = 84, r = 56, k = 6, \lambda = 35$
26	$v = 10, k = 8$	$v = 9, b = 36, r = 28, k = 7, \lambda = 21$
27	$v = 10, k = 9$	$v = 9, b = 9, r = 8, k = 8, \lambda = 7$
28	$v = 11, k = 6$	$v = 10, b = 252, r = 18, k = 5, \lambda = 56$
29	$v = 11, k = 7$	$v = 10, b = 210, r = 18, k = 6, \lambda = 70$
30	$v = 11, k = 8$	$v = 10, b = 120, r = 18, k = 7, \lambda = 56$

4. Conclusion

In conclusion, the study established that for un-reduced BIBD with parameters v and $k \geq 3$. If blocks from the design containing treatment i are extracted from the design and in the extracted blocks, treatment i is deleted from all the blocks, leaving the rest of the treatments, then a BIBD is formed with parameters $v^* = v - 1, b^* = \binom{v-1}{k-1}, k^* = k - 1, r^* = \binom{v-2}{k-2}, \lambda = \binom{v-3}{k-3}$. This class of BIBD is known as Derived Reduced Balanced Incomplete Block Design. The study was, therefore, able to derive a new technique for constructing BIBDs from existing BIBDs, which adds to the list of methods used in designing BIBDs.

Competing Interests

Authors have declared that no competing interests exist.

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