Mountain Atmospherics

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ABSTRACT

The purpose of the work is to examine the effects of compressibility on air properties when a wind blows against a sloping mountain surface. Previous research of air compression effects include the low speed wing and the crests of surface gravity waves propagating in the wind. In both cases, an algebraic expression was obtained for the lift force. When wind blows across a mountain and the assumption is made that a boundary layer of compressed air forms and remains attached to the mountain, a physical-chemical theory predicts that the wind will have no shear and the pressure and density will decrease with increasing altitude at the same rate. Combining Bernoulli's law along streamlines with the cross-stream force balance, pressure gradient equals centrifugal force, and the perfect gas law for air, is the model used here.

1. INTRODUCTION

Consider a mountain range rising up out of a level plain with a horizontal wind field blowing toward it in a two-dimensional configuration. When the moving air interacts with the solid sloping surface of the mountain, it is proposed that there forms a boundary layer attached to the mountain containing compressed air with a higher density than that in the environment. This suggestion is made by analogy with a much smaller scale problem: the compressed air boundary layer on a low speed wing calculated before [1]. Also the lift force which makes crests of the surface gravity waves grow during propagation through the wind has been analyzed [2]. Finally, the lift force acting on equatorial sea level, due to the dynamics of the Trade Wind boundary layers, has been estimated [3].

After the boundary layer characteristics for the mountain are worked through, it turns out that the vertical profiles above the mountain for both pressure and density have the same form: they decrease with increasing altitude as constant times the inverse square of the radius from the center of the arc that approximates the shape of the mountain top.

These results do not fit in with in with what one usually understands by the concept of the atmosphere's scale-height, but that subject has been accompanied by confusion [4].

2. METHOD

On streamlines going over the mountain Bernoulli's law holds

$$p = C - \frac{1}{2}\rho u^2 \qquad (1)$$

The pressure is p, the density is ρ and the flow speed is u. It is assumed that the constant C is the same for all streamlines, and if $C = 0$, there will be no significant modification of the dynamics.

Across streamlines for a steady state there is a balance of two opposing forces: the upward centrifugal force, where the streamlines are curved, and a downward pressure gradient

$$\frac{\delta p}{\delta r} = \frac{\rho u^2}{r} \qquad (2)$$

And r is measured from the center of the streamline curvature.

Finally, for air as a perfect gas the equation of state is

$$p = \rho R T \qquad (3)$$

where R is he gas constant and T the temperature.

By eliminating u and ρ among Equations (1) - (3) a single first order differential equation in p can be obtained

$$r\frac{\partial p}{\partial r} + 2p = 0 \qquad (4)$$

which has the solution

$$p = \frac{const}{r^2} \qquad (5)$$

where evaluation of the constant is coming below in (6).

Comparing (5) and (3) shows that the density has the same general form as the pressure, which is a prediction involving the inverse square of the altitude. Then from (1) it can be inferred that the wind speed u above the mountain cannot be a function of r, *i.e.* there is no shear in the wind, another prediction.

3. DISCUSSION

Conserving mass flux across the mountain between a vertical section at the top and a vertical section on the plain in front of the mountain leads to the following form of the constant in (5)

$$const = \rho_0 h r_0 S \qquad (6)$$

where ρ_0 is the environmental density, h is the greatest mountain height above the plain, r_0 is the radius of curvature of the mountain top and

$$S = RT \qquad (7)$$

where R is the gas constant for air and T is the temperature.

Equation (6) is directly analogous to what was found in working out the lift force on a circular arc wing [1].

Three predictions given above involving the variations with altitude of the wind speed, air pressure and density need to be verified by measurements in the future.

4. CONCLUSION

Based on a model of a compressed air boundary layer attached to a mountain with wind blowing over it, predictions are that there is no shear in the wind and that both pressure and density decrease with in-

crease in altitude at the same rate, which is as the inverse square of the radius of the arc that approximates the shape of the mountain top.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

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