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Four-Step One Hybrid Block Methods for Solution of Fourth Derivative Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration among all authors. Author RD designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors SY and AL managed the analyses of the study. Author AL managed the literature searches. All authors read and approved the final manuscript.

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Abstract

We consider developing a four-step one offgrid block hybrid method for the solution of fourth derivative Ordinary Differential Equations. Method of interpolation and collocation of power series approximate solution was used as the basis function to generate the continuous hybrid linear multistep method, which was then evaluated at non-interpolating points to give a continuous block method. The discrete block method was recovered when the continuous block was evaluated at all step points. The basic properties of the methods were investigated and said to be converge. The developed four-step method is applied to solve fourth derivative problems of ordinary differential equations from the numerical results obtained; it is observed that the developed method gives better approximation than the existing method compared with.

Keywords: Four-step; hybrid point; fourth derivative; power series; ODE's; interpolation.

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1 Introduction

In this paper, a four-step one off grid point hybrid block method is considered to approximate ordinary differential equations of the form

$$y^{iv} = f(x, y, y', y'', y'''), \quad y(t_0) = y_0,\ y'(t_0) = y'_0,\ y''(t_0) = y''_0,\ y'''(t_0) = y'''_0 \tag{1}$$

Various approaches can be used for the analytic solutions of fourth order ordinary differential equations. Researchers are interested in equation (1) because of its wide area of applications in various fields such as in modeling scientific and engineering, control theory, fluid dynamics, mechanical systems without dissipation, celestial mechanics and other related real life problems.

Solving higher order derivatives method by reducing them to a system of first-derivative approach involves more functions to evaluate which then leads to a computational burden as in [1,2]. Different method have been proposed for the solution of (1) ranging from predictor-corrector method to hybrid methods. Despite the success recorded by the predictor-corrector methods, its major setback is that the predictor are in reducing order of accuracy especially when the value of the step-length is high and moreover the result are at overlapping interval. However, many researchers have addressed these setbacks [3,4,5,6,7,8]. The direct methods of solving (1) as reported in Literatures is more efficient and gives high accuracy and speed than the method of reduction to first order ordinary differential equations [9,10,11,12, 13,14].

Scholars who recently adopted the hybrid method other than the direct method in approximation of (1) include among others [15,16,17].

In this paper, we developed a four-step one offgrid hybrid point block method for solution of initial value problems of fourth order ordinary differential equation, which is implemented in block. The method developed evaluates less function per step and circumventing the Dahlquist barrier's by the introduction of a hybrid points.

The paper is organised as follows: In section 2, we discuss the methods and the materials for the development of the method. Section 3 considers analysis of the basis properties of the method, numerical experiments where the efficiency of the derived method is demonstrated on some numerical examples and discussion of results. Lastly, we concluded in section 4.

2 Derivation of the Method

This section describes the objective of which the derivations of the hybrid block method using the linear multistep Algorithm. The Algorithm shall be in the form

$$y(x) = \sum_{i=0}^{3} \alpha_i y_{n+i} + h^4 \left[\sum_{j=0}^{4} \beta_j f_{n+j} + \beta_k f_{n+k} \right], k = \frac{1}{2} \tag{2}$$

$\alpha_i(t)$, $\beta_j(t)$, $\beta_k(t)$ are polynomials, $y_{n+j} = y\left(x_{n+j}\right), f_{n+j} = f\left(x_{n+j}, y_{n+j}\right)$ $t = \frac{x - x_n}{h}$

On the partition $[a,b]$, where α_0 *and* β_0 are non zero.

Equation (2) is obtained by considering the approximate solution of the power series in form of

$$y(x) = \sum_{j=0}^{s+r-1} a_i \left(\frac{x - x_n}{h}\right)^j \tag{3}$$

$r=3$ and $s=3$ are the numbers of interpolation and collocation points. The continuouos approximation is then constructed by imposing two conditions which are

$$\left.\begin{aligned} y_{n+j} &= y\left(x_{n+j}\right),\ j=0,1,2,3 \\ y^{iv}\left(x_{n+j}\right) &= f_{n+j} \end{aligned}\right\} \tag{4}$$

Equation (4) result to $(r+s)$, which gives a non linear equation of the form

$$AX = U \tag{5}$$

Which will then be evaluated through a matrix inversion algorithm in which the values of α_i*'s and* β_j*'s* are determined. By the substitutions of the values of α_i*'s and* β_j*'s* obtained into equation (3) gives a continuous hybrid linear multistep method of the form

$$y(x) = \sum_{i=0}^{3} \alpha_i(x) y_{n+i} + h^4 \left[\sum_{j=0}^{4} \beta_j f_{n+j} + \beta_k f_{n+k}\right], k = \frac{1}{2} \tag{6}$$

We then impose (4) on $y(x)$ in (3) and the coefficient of $y_{n+i}, i=0,1,,2,3$ and $f_{n+j}, j=0,1,2,3,4,\frac{1}{2}$ give

$$y_{n+t} = \sum_{i=0}^{3}\left(\alpha_i(t) y_{n+i}\right) + h^4\left[\beta_0(t) f_n + \beta_1(t) f_{n+1} + \beta_2(t) f_{n+2} + \beta_3(t) f_{n+3} + \beta_4(t) f_{n+4} + \beta_{\frac{1}{2}}(t) f_{n+\frac{1}{2}}\right] \tag{7}$$

Where $t = \frac{x - x_{n+4}}{h}, \frac{dt}{dx} = \frac{1}{h}$

$$\alpha_0 = 1 - \frac{11}{6}\frac{-x_n + x}{h} + \frac{(-x_n + x)^2}{h^2} - \frac{1}{6}\frac{(-x_n + x)^3}{h^3} \quad \alpha_1 = \frac{3(-x_n + x)}{h} - \frac{5}{2}\frac{(-x_n + x)^2}{h^2} + \frac{1}{2}\frac{(-x_n + x)^3}{h^3}$$

$$\alpha_2 = -\frac{3}{2}\frac{-x_n + x}{h} + \frac{2(-x_n + x)^2}{h^2} - \frac{1}{2}\frac{(-x_n + x)^3}{h^3} : \quad \alpha_3 = \frac{1}{3}\frac{-x_n + x}{h} - \frac{1}{2}\frac{(-x_n + x)^2}{h^2} + \frac{1}{6}\frac{(-x_n + x)^3}{h^3} :$$

$$\beta_0 = -\frac{137}{20160}(-x_n + x)h^3 + \frac{103}{5760}(-x_n + x)^2 h^2 - \frac{2801}{90720}(-x_n + x)^3 h + \frac{1}{24}(-x_n + x)^4 - \frac{49}{1440}\frac{(-x_n + x)^5}{h}$$
$$+ \frac{1}{64}\frac{(-x_n + x)^6}{h^2} - \frac{1}{252}\frac{(-x_n + x)^7}{h^3} + \frac{1}{1920}\frac{(-x_n + x)^8}{h^4} - \frac{1}{36288}\frac{(-x_n + x)^9}{h^5}$$

$$\beta_{\frac{1}{2}} = -\frac{16}{1575}(-x_n + x)h^3 + \frac{92}{1323}(-x_n + x)^2 h^2 - \frac{1766}{19845}(-x_n + x)^3 h + \frac{32}{525}\frac{(-x_n + x)^5}{h} - \frac{8}{189}\frac{(-x_n + x)^6}{h^2}$$

$$+\frac{4}{315}\frac{(-x_n+x)^7}{h^3}-\frac{4}{2205}\frac{(-x_n+x)^8}{h^4}+\frac{2}{19845}\frac{(-x_n+x)^9}{h^5}$$

$$\beta_1=-\frac{283}{1680}(-x_n+x)h^3+\frac{8231}{30240}(-x_n+x)^2h^2-\frac{4279}{45360}(-x_n+x)^3h-\frac{1}{30}\frac{(-x_n+x)^5}{h}+\frac{37}{1080}\frac{(-x_n+x)^6}{h^2}$$
$$-\frac{61}{5040}\frac{(-x_n+x)^7}{h^3}+\frac{19}{10080}\frac{(-x_n+x)^8}{h^4}-\frac{1}{9072}\frac{(-x_n+x)^9}{h^5}$$

$$\beta_2=-\frac{683}{10080}(-x_n+x)h^3+\frac{6281}{60480}(-x_n+x)^2h^2-\frac{347}{9072}(-x_n+x)^3h+\frac{1}{120}\frac{(-x_n+x)^5}{h}-\frac{43}{4320}\frac{(-x_n+x)^6}{h^2}$$
$$+\frac{23}{5040}\frac{(-x_n+x)^7}{h^3}-\frac{17}{20160}\frac{(-x_n+x)^8}{h^4}+\frac{1}{18144}\frac{(-x_n+x)^9}{h^5}$$

$$\beta_3=\frac{13}{3600}(-x_n+x)h^3-\frac{59}{10080}(-x_n+x)^2h^2+\frac{127}{45360}(-x_n+x)^3h-\frac{1}{450}\frac{(-x_n+x)^5}{h}+\frac{1}{360}\frac{(-x_n+x)^6}{h^2}$$
$$-\frac{1}{720}\frac{(-x_n+x)^7}{h^3}+\frac{1}{3360}\frac{(-x_n+x)^8}{h^4}-\frac{1}{45360}\frac{(-x_n+x)^9}{h^5}$$

$$\beta_4=-\frac{1}{2240}(-x_n+x)h^3+\frac{613}{846720}(-x_n+x)^2h^2-\frac{223}{635040}(-x_n+x)^3h+\frac{1}{3360}\frac{(-x_n+x)^5}{h}$$
$$-\frac{23}{60480}\frac{(-x_n+x)^6}{h^2}+\frac{1}{5040}\frac{(-x_n+x)^7}{h^3}-\frac{13}{282240}\frac{(-x_n+x)^8}{h^4}+\frac{1}{254016}\frac{(-x_n+x)^9}{h^5}:$$

The first, second and third derivatives of (6) gives

$$y'(x)=\frac{1}{h}\left(\sum_{i=0}^{3}\alpha'_i(x)y_{n+i}+h^4\left[\sum_{j=0}^{4}\beta'_j f_{n+j}+\beta'_k f_{n+k}\right], k=\frac{1}{2}\right) \tag{8}$$

$$y''(x)=\frac{1}{h^2}\left(\sum_{i=0}^{3}\alpha''_i(x)y_{n+i}+h^4\left[\sum_{j=0}^{4}\beta''_j f_{n+j}+\beta''_k f_{n+k}\right], k=\frac{1}{2}\right) \tag{9}$$

$$y'''(x)=\frac{1}{h^3}\left(\sum_{i=0}^{3}\alpha'''_i(x)y_{n+i}+h^4\left[\sum_{j=0}^{4}\beta'''_j f_{n+j}+\beta'''_k f_{n+k}\right], k=\frac{1}{2}\right) \tag{10}$$

We use equation (7) at $x=x_{n+\frac{1}{2}}, x=x_{n+4}$ to get

$$y_{n+\frac{1}{2}}=\frac{5}{16}y_n+\frac{15}{16}y_{n+1}-\frac{5}{16}y_{n+2}+\frac{1}{16}y_{n+3}$$
$$-\frac{1}{184320}h^4\left(190f_n-464f_{n+\frac{1}{2}}+5265f_{n+1}+2315f_{n+2}-121f_{n+3}+15f_{n+4}\right) \tag{11}$$

$$y_{n+4}=-y_n+4y_{n+1}-6y_{n+2}+4y_{n+3}-\frac{1}{720}h^4\left(f_n-124f_{n+1}-474f_{n+2}-124f_{n+3}+f_{n+4}\right) \tag{12}$$

Evaluating (8), (9) and (10) at all points we obtain equations (13), (14) and (15) as shown in Tables 1, 2 and 3 respectively.

Table 1. Coefficients of $\alpha'_i{'s}$ *and* $\beta'_j{'s}$ for equation (8) which was evaluated at all points gives

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	f_{n+2}	f_{n+3}	f_{n+4}
t_n	$-\frac{11}{6}$	3	$-\frac{3}{2}$	$\frac{1}{3}$	$-\frac{137}{20160}$	$-\frac{16}{1575}$	$-\frac{283}{1680}$	$-\frac{683}{10080}$	$\frac{13}{3600}$	$-\frac{1}{2240}$
$t_{n+\frac{1}{2}}$	$-\frac{23}{24}$	$\frac{7}{8}$	$\frac{1}{8}$	$-\frac{1}{24}$	$\frac{10169}{15482880}$	$\frac{3041}{604800}$	$\frac{569}{20480}$	$\frac{66527}{7741440}$	$-\frac{8707}{19353600}$	$\frac{283}{5160960}$
t_{n+1}	$-\frac{1}{3}$	$-\frac{1}{2}$	1	$-\frac{1}{6}$	$\frac{167}{60480}$	$-\frac{44}{3675}$	$\frac{185}{3024}$	$\frac{331}{10080}$	$-\frac{127}{75600}$	$\frac{89}{423360}$
t_{n+2}	$\frac{1}{6}$	-1	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{157}{60480}$	$\frac{424}{33075}$	$-\frac{47}{1008}$	$-\frac{1447}{30240}$	$\frac{83}{75600}$	$-\frac{5}{28224}$
t_{n+3}	$-\frac{1}{3}$	$\frac{3}{2}$	-3	$\frac{11}{6}$	$\frac{43}{3600}$	$-\frac{44}{3675}$	$\frac{377}{5040}$	$\frac{583}{3360}$	$\frac{43}{3600}$	$-\frac{59}{141120}$
t_{n+4}	$-\frac{11}{6}$	7	$-\frac{19}{2}$	$\frac{13}{3}$	$\frac{17}{60480}$	$-\frac{16}{1575}$	$\frac{703}{2160}$	$\frac{12737}{10080}$	$\frac{37613}{75600}$	$\frac{401}{60480}$

(13)

Table 2. Coefficients of $\alpha''_i{'s}$ *and* $\beta''_j{'s}$ for equation (9) which was evaluated at all points gives

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	f_{n+2}	f_{n+3}	f_{n+4}
t_n	2	-5	4	-1	$\frac{103}{2880}$	$\frac{184}{1323}$	$\frac{8231}{15120}$	$\frac{6281}{30240}$	$-\frac{59}{5040}$	$\frac{613}{423360}$
$t_{n+\frac{1}{2}}$	$\frac{3}{2}$	$-\frac{7}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{919}{120960}$	$-\frac{701}{17640}$	$\frac{7879}{34560}$	$\frac{2699}{26880}$	$-\frac{253}{48384}$	$\frac{1097}{1693440}$
t_{n+1}	1	-2	1	0	$-\frac{47}{60480}$	$-\frac{44}{6615}$	$-\frac{359}{5040}$	$-\frac{23}{4320}$	$\frac{11}{15120}$	$-\frac{11}{141120}$
t_{n+2}	0	1	-2	1	$\frac{11}{60480}$	0	$-\frac{53}{15120}$	$-\frac{773}{10080}$	$-\frac{53}{15120}$	$\frac{11}{60480}$
t_{n+3}	-1	4	-5	2	$-\frac{61}{20160}$	$-\frac{44}{6615}$	$\frac{2483}{15120}$	$\frac{19967}{30240}$	$\frac{31}{336}$	$-\frac{1577}{423360}$
t_{n+4}	-2	7	-8	3	$\frac{1891}{60480}$	$-\frac{184}{1323}$	$\frac{371}{720}$	$\frac{41497}{30240}$	$\frac{16199}{15120}$	$\frac{617}{9408}$

(14)

Table 3. Coefficients of $\alpha'''_i{'s}$ *and* $\beta'''_j{'s}$ for equation (10) which was evaluated at all points gives

t	y_n	y_{n+1}	y_{n+2}	y_{n+3}	f_n	$f_{n+1/2}$	f_{n+1}	f_{n+2}	f_{n+3}	f_{n+4}
t_n	-1	3	-3	1	$-\frac{2801}{15120}$	$-\frac{3532}{6615}$	$-\frac{4279}{7560}$	$-\frac{347}{1512}$	$\frac{127}{7560}$	$-\frac{223}{105840}$
$t_{n+\frac{1}{2}}$	-1	3	-3	1	$-\frac{3931}{483840}$	$-\frac{2809}{26460}$	$-\frac{8371}{12096}$	$-\frac{48947}{241920}$	$\frac{149}{15120}$	$-\frac{4049}{3386880}$
t_{n+1}	-1	3	-3	1	$-\frac{73}{3024}$	$\frac{1004}{6615}$	$-\frac{1583}{3780}$	$-\frac{209}{945}$	$\frac{53}{3780}$	$-\frac{181}{105840}$
t_{n+2}	-1	3	-3	1	$\frac{391}{15120}$	$-\frac{844}{6615}$	$\frac{2777}{7560}$	$\frac{1961}{7560}$	$-\frac{209}{7560}$	$\frac{281}{105840}$
t_{n+3}	-1	3	-3	1	$-\frac{533}{15120}$	$\frac{1004}{6615}$	$-\frac{31}{756}$	$\frac{988}{945}$	$\frac{1481}{3780}$	$-\frac{1357}{105840}$
t_{n+4}	-1	3	-3	1	$\frac{1903}{15120}$	$-\frac{3532}{6615}$	$\frac{6473}{7560}$	$\frac{2297}{7560}$	$\frac{10879}{7560}$	$\frac{6541}{21168}$

(15)

3 Analysis of the Method

3.1 Order of the block

According to fatunla (1991) and lambert (1973) the truncation error associated with (2) is defined by

$$L[y(x);h]=\sum_{i=0}^{3}\left(\alpha_i(t)y_{n+i}\right)-h^4\beta_0 y^{iv}(x+jh)-h^4\beta_1 y^{iv}(x+h)-h^4\beta_2(y^{iv}(x+2h)-h^4\beta_3 y^{iv}(x+3h)-h^4\beta_4 y^{iv}(x+4h)-h^4\beta_{\frac{1}{2}}y^{iv}\left(x+\frac{1}{2}h\right) \quad (16)$$

Assumed that $y(x)$ can be differentiated. Expanding (16) in Taylor's series and comparing the coefficient of h gives the expression

$$L\{y(x):h\}=C_0y(x)+C_1y'(x)+,\ldots+C_ph^py^p(x)+C_{p+1}h^{p+1}y^{p+1}(x)+C_{p+2}h^{p+2}y^{p+2}(x)+C_{p+3}h^{p+3}y^{p+3}(x)+$$

Where the constant coefficients are given below

$$C_0=\sum_{j=0}^{k}\alpha_j \ , \ C_1=\sum_{j=1}^{k}j\alpha_j$$

$$C_q=\frac{1}{q!}\sum_{j=0}^{k}j^q\alpha_j-q(q-1)(q-2)(q-3)\left(\sum_{j=0}j^{q-4}\beta_j+1^{q-4}\beta_1+2^{q-4}\beta_2+3^{q-4}\beta_3+4^{q-4}\beta_4+\left(\frac{1}{2}\right)^{q-4}\beta_{\frac{1}{2}}\right),\quad q=2,3,4...$$

Definition 1: The linear operator and the associated continuous linear multistep method (5) are said to be of order p if $c_0=c_1=c_2=...=c_p=0$ $c_{p+1}=0, c_{p+2}=c_{p+3}=0.\,and\ c_{p+4}\neq 0,$ c_{p+4} is called the error constant and the local truncation error is given by

$$t_{n+k}=c_{p+4}h^{(p+4)}y^{(p+4)}(x_n)+o\left(h^{p+5}\right) \quad (17)$$

For our method

Comparing the coefficient of h gives $C_0=C_1=C_2=C_3=...=C_6=0$ and

$$C_7=\left[-\frac{311}{774144},-\frac{5}{24192},-\frac{11}{7560},\frac{1}{896},-\frac{8}{945}\right]^T$$

Hence our method is of order three (3).

3.2 Consistency

Four-Step One Hybrid Block fourth derivative hybrid method is said to be consistent according to Areo and Omojola (2015) if all the following six conditions are satisfied

The order of the method must be greater than or equal to one i.e $(p \geq 1)$

i. $\sum_{j=0}^{k} \alpha_j = 0$ and α_j's are the coefficients of the first characteristics polynomial $\rho(r)$

ii. $\rho(r) = \rho'(r) = 0$ for $r = 1$

iii. $\rho''(r) = 2!\sigma(r)$ for $r = 1$

iv. $\rho'''(r) = 3!\sigma(r)$ for $r = 1$

v. $\rho^{iv}(r) = 4!\sigma(r)$ for $r = 1$

3.3 Zero stability of our method

Four-Step One Hybrid Block fourth derivative hybrid method is said to be zero-stable if as $h \to 0$, the root $z_i, i = 1(1)k$ of the first characteristic polynomial $\rho(z) = 0$ that is $\rho(z) = \det\left[\sum_{j=0}^{k} A^{(i)} z^{k-i}\right] = 0$ Satisfies $|z_i| \leq 1$ and for those roots with $|z_i| = 1$, multiplicity must not exceed two.

Hence, our method is zero-stable.

3.4 Numerical example

Problem I We consider a special fourth order differential equation (Source: Adoghe & Omole 2019)

$y^{iv} = -\sin x + \cos x$, $y(0) = 0$, $y'(0) = -1$, $y''(0) = -1$, $y''(0) = 7$

Exact Solution: $y(x) = -\sin x + \cos x + x^3 - 1$, $h = \frac{1}{320}$

Table 4. Comparison of the proposed method with Adoghe and Omole 2019

x-values	Exact solution	Computed solution	Error in our method	Error in [17]
0.003125	- 0.00312984720468769600	- 0.00312984720468769600	0.0000e+00	5.8350e-18
0.00625	- 0.00626924635577210114	- 0.00626924635577210114	0.0000e+00	4.6708e-17
0.009375	- 0.00941798368752841945	- 0.00941798368752841944	1.0000e-20	5.2467e-17
0.00125	- 0.01257584533946248273	- 0.01257584533946248273	0.0000e+00	9.3430e-17
0.015625	- 0.01574261735661109244	- 0.01574261735661109244	0.0000e+00	9.9220e-17
0.01875	- 0.01891808568984328399	- 0.01891808568984328399	0.0000e+00	1.4019e-16
0.021875	- 0.02210203619616251069	- 0.02210203619616251070	1.0000e-20	1.4613e-16
0.025	- 0.02529425463900974441	- 0.02529425463900974442	1.0000e-20	1.8712e-16
0.028125	- 0.02849452668856748983	- 0.02849452668856748984	1.0000e-20	1.9324e-16
0.03125	- 0.03170263792206470950	- 0.03170263792206470951	1.0000e-20	5.8350e-18

Problem II We consider the fourth order ODE (Source: Akinfenwa et al. 2016)

$y^{iv} = 4y''$, $y(0) = 1, y'(0) = 3, y''(0) = 0, y'''(0) = 16$

Exact Solution: $y(x) = 1 - x + \exp(2x) - \exp(-2x)$, $h = \frac{1}{320}$

Table 5. Comparison of the proposed method with Akinfenwa et al 2016

x-values	Exact solution	Computed solution	Error in our method	Error in [16]
0.003125	1.00937508138036727920	1.00937508138036727920	0.00e+00	1.00e-18
0.00625	1.01875065104675294860	1.01875065104675294860	0.00e+00	2.00e-18
0.009375	1.02812719730424913310	1.02812719730424913310	0.00e+00	5.20e-17
0.00125	1.03750520849609617210	1.03750520849609617200	1.00e-19	2.39e-16
0.015625	1.04688517302275858900	1.04688517302275858910	1.00e-19	5.52e-16
0.01875	1.05626757936100329750	1.05626757936100329750	0.00e+00	9.57e-16
0.021875	1.06565291608298078600	1.06565291608298078600	0.00e+00	1.20e-15
0.025	1.07504167187531003060	1.07504167187531003060	0.00e+00	1.21e-15
0.028125	1.08443433555816787740	1.08443433555816787740	0.00e+00	6.27e-16
0.03125	1.09383139610438364350	1.09383139610438364340	1.00e-19	5.54e-16

Problem III Consider the initial value problem (source: Adeyeye & Omar 2018)

$$y^{iv} = -y'', \quad y(0) = 0, \; y'(0) = -\frac{1.1}{72-50\pi}, \; y''(0) = \frac{1}{144-100\pi}, \; y'''(0) = \frac{1.2}{144-100\pi}$$

Exact Solution: $y(x) = \frac{1-x-\cos x-1.2\sin x}{144-100\pi}$ *with* $h = \frac{1}{10}$

Table 6. Comparison of the proposed method with Adeyeye & Omar 2018

x-values	Exact solution	Computed solution	Error in our method	Error in [7]
0.1	0.00004034461209373069	0.00004034461209373069	0.00e+00	6.51e-19
0.2	0.00008063166098895974	0.00008063166098895974	0.00e+00	1.30e-18
0.3	0.00012086093247161511	0.00012086093247161511	0.00e+00	4.77e-18
0.4	0.00016103221289185685	0.00016103221289185685	0.00e+00	1.73e-17
0.5	0.00020114528916616351	0.00020114528916616351	0.00e+00	4.34e-17
0.6	0.00024119994877941305	0.00024119994877941305	0.00e+00	9.54e-17
0.7	0.00028119597978695816	0.00028119597978695816	0.00e+00	1.81e-16
0.8	0.00032113317081669604	0.00032113317081669604	0.00e+00	3.16e-16
0.9	0.00036101131107113260	0.00036101131107113259	1.00e-20	5.19e-16
1.0	0.00040083019032944098	0.00040083019032944098	0.00e+00	8.05e-16

4 Conclusions

It is evident from the above tables that our proposed method has significant improvement over the existing methods. The four-step one hybrid point block method is proposed for direct solution of general fourth order ordinary differential equations where by it is self-starting when implemented. The developed method converges and is of Order three.

Competing Interests

Authors have declared that no competing interests exist.

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