Asian Research Journal of Mathematics

17(11): 1-6, 2021; Article no.ARJOM.78399
ISSN: 2456-477X

A New Inequality with Its Application in Solving a Problem of Inequalities

Yuanjie Guo [a] and Kongfeng Zhu [b*]

[a] Department of Mechatronic Engineering, Foshan University, Foshan City, China.
[b] Guangzhou CNC Equipment Co. LTD, Guangzhou, 510000, China.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2021/v17i1130338

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/78399

Original Research Article

Received 06 October 2021
Accepted 13 December 2021
Published 14 December 2021

Abstract

This article first puts forwards and proves a new inequality, then use the inequality to solve a problem related with a series of inequalities. Detail mathematical reasoning and proofs are presented. The results are valuable to learn and research inequalities.

Keywords: Inequality; sort; mathematical reasoning; ascending order.

1 Introduction

Recent research about distribution of the positive integers in T_3 tree, as introduced in [1-6], has come across a problem of putting a sequence of numbers in their ascending order. The numbers are $\frac{2}{y+1}$, $\frac{2}{\alpha+1}$, $\sqrt{\frac{1}{y}}$, $\frac{2}{x+1}$, $\sqrt{\frac{1}{\alpha}}$, $\sqrt{\frac{1}{x}}$, $\frac{2\alpha}{\alpha+1}$, $\sqrt{x}$, $\frac{2y}{y+1}$, $\frac{x+1}{2}$, $\sqrt{\alpha}$ and $\sqrt{y}$. The problem involves in proving a series

**Corresponding author: Email: gskzkf@163.com;*

of the inequalities. Look into the reference handbooks [7-11] no referable references were found. Thereby this paper investigates the problem and finds out a solution.

2 Preliminaries

2.1 Symbols and Notations

Symbol $A \otimes B$ means A holds and simultaneously B holds. Symbol $A \Rightarrow B$ means conclusion B can be derived from condition A.

2.2 Lemma 1

(See in [12]. Let α be a real number with $\alpha \in (1,4) \cup (4,\infty)$; then

$$f(\alpha) = \frac{\sqrt{\alpha}}{2-\sqrt{\alpha}} - (\frac{\alpha+1}{2})^2 .$$

Thus

$$0 < f(\alpha) < \infty, \ \alpha \in (1,4)$$

and

$$-\infty < f(\alpha) < (-17.345), \ \alpha \in (4,6) .$$

3 Main Result and Proof

Theorem1. Let α be a real number with $a \in (1,4)$; then

$$f(\alpha) = 2\sqrt{\alpha} - 1 - (\frac{2\alpha}{\alpha+1})^2$$

thus

$$0 < f(\alpha) < 0.44, \ \alpha \in (1,4) .$$

Proof. Let $f(\alpha) = 2\sqrt{\alpha} - 1 - (\frac{2\alpha}{\alpha+1})^2$, $\alpha \in (1,4)$.

Simplify the function

$$f(\alpha) = \frac{(2\sqrt{\alpha}-1)(\alpha+1)^2 - 4\alpha^2}{(\alpha+1)^2} .$$

The denominator is greater than zero obviously.

Assume that α+1 = t; then $2 < t < 5$,thus

$$f(\alpha) = (2\sqrt{t-1} - 1) \times t^2 - 4(t-1)^2 .$$

Let $h(t)=(2\sqrt{t-1}-1)\times t^2-4(t-1)^2$; then

$$h^{'}(t)=\frac{4t(t+1)+t^2}{\sqrt{t-1}}-10t+8.$$

When $t\in(2,5)$, it obviously holds

$$12<\frac{4t(t+1)+t^2}{\sqrt{t-1}}<52.5$$

and

$$12<10t-8<42.$$

Direct calculation yields

$$0<h^{'}(t)<10.5.$$

This means $h(t)$ is monotonically increasing in the condition of $t\in(2,5)$. Under the condition $h(2)=0$ and $0<h^{'}(t)<10.5$ when $t\in(2,5)$, it is obtained $h(t)>0$. Meanwhile, this conclusion illustrates $f(\alpha)>0$ if $\alpha\in(1,4)$.

Theorem2. Let $1<\alpha<4$, x and y satisfy

$$1<2\sqrt{\alpha}-1\le x\le(\frac{2y}{y+1})^2\le\alpha\le y\le(\frac{\alpha+1}{2})^2<4 \tag{1}$$

then

$$\frac{2}{w+1}\le\frac{2}{y+1}\le\frac{2}{\alpha+1}\le\sqrt{\frac{1}{y}}\le\frac{2}{x+1}\le\sqrt{\frac{1}{\alpha}}\le\sqrt{\frac{1}{x}}\le1\le\frac{2\alpha}{\alpha+1}\le\sqrt{x}\le\frac{2y}{y+1}\le\sqrt{\alpha}\le\frac{x+1}{2}\le\sqrt{y}\le\sqrt{w}$$

Proof. (1) $u\le x\le\alpha\le y\le w\Rightarrow\sqrt{u}\le\sqrt{x}\le\sqrt{\alpha}\le\sqrt{y}\le\sqrt{w}$.

$$x\ge2\sqrt{\alpha}-1\Rightarrow\frac{x+1}{2}\ge\sqrt{\alpha} \tag{2}$$

$$\frac{(\frac{2\alpha}{\alpha+1})^2}{\alpha}=\frac{4\alpha}{(\alpha+1)^2}\le\frac{4\alpha}{4\alpha}=1\Rightarrow\frac{2\alpha}{\alpha+1}\le\sqrt{\alpha} \tag{3}$$

$$\frac{y}{y+1}-\frac{\alpha}{\alpha+1}=\frac{y(\alpha+1)-\alpha(y+1)}{(y+1)(\alpha+1)}=\frac{y-\alpha}{(y+1)(\alpha+1)}\ge0\Rightarrow\frac{2y}{y+1}\ge\frac{2\alpha}{\alpha+1} \tag{4}$$

$$\frac{x+1}{2}-\frac{2y}{y+1}=\frac{(x+1)(y+1)-2y}{2(y+1)}=\frac{(x-1)y+x+1}{2(y+1)}\ge0\Rightarrow\frac{x+1}{2}\ge\frac{2y}{y+1} \tag{5}$$

By Lemma 1, $f(\alpha)=\frac{\sqrt{\alpha}}{2-\sqrt{\alpha}}-(\frac{\alpha+1}{2})^2>0,\alpha\in(1,4)$, it immediately leads to

$$\frac{\sqrt{\alpha}}{2-\sqrt{\alpha}} \geq (\frac{\alpha+1}{2})^2 \geq y \Rightarrow 2y - y\sqrt{\alpha} \leq \sqrt{\alpha} \Rightarrow \frac{2y}{y+1} \leq \sqrt{\alpha} \tag{6}$$

By Theorem 1, $f(\alpha) = 2\sqrt{\alpha} - 1 - (\frac{2\alpha}{\alpha+1})^2 > 0, \alpha \in (1,4)$. $2\sqrt{\alpha} - 1 \geq (\frac{2\alpha}{\alpha+1})^2$ and $x > 2\sqrt{\alpha} - 1$ result in

$$\sqrt{x} \geq \frac{2\alpha}{\alpha+1} \tag{7}$$

By Theorem 1, it holds

$$2\sqrt{y} - 1 > (\frac{2y}{y+1})^2, y \in (1,4) \tag{8}$$

Therefore, $x \leq (\frac{2y}{y+1})^2 \otimes (\frac{2y}{y+1})^2 \leq 2\sqrt{y} - 1 \Rightarrow \frac{x+1}{2} \leq \sqrt{y}$.

$$\sqrt{x} \leq \frac{2y}{y+1} \text{ is from given condition} \tag{9}$$

$$w \geq y \geq \alpha \geq x \Rightarrow y+1 \geq \alpha+1 \geq x+1 \Rightarrow \frac{2}{x+1} \geq \frac{2}{\alpha+1} \geq \frac{2}{y+1} \geq \frac{2}{w+1} \tag{10}$$

$$y \geq \alpha \geq x \Rightarrow \sqrt{\frac{1}{x}} \geq \sqrt{\frac{1}{\alpha}} \geq \sqrt{\frac{1}{y}} \tag{11}$$

$$\frac{x+1}{2} \geq \sqrt{\alpha} \Rightarrow \frac{1}{x+2} \leq \sqrt{\frac{1}{\alpha}} \tag{12}$$

$$y \leq (\frac{\alpha+1}{2})^2 \Rightarrow \sqrt{y} \leq \frac{\alpha+1}{2} \Rightarrow \frac{2}{\alpha+1} \leq \sqrt{\frac{1}{y}} \tag{13}$$

$$x \leq (\frac{2y}{y+1})^2 \leq 2\sqrt{y} - 1 \Rightarrow \frac{x+1}{2} \leq \sqrt{y} \Rightarrow \sqrt{\frac{1}{y}} \leq \frac{2}{x+1} \text{ (the proof is similar to (8)).} \tag{14}$$

Accordingly, it holds

$$\frac{2}{y+1} \leq \frac{2}{\alpha+1} \leq \sqrt{\frac{1}{y}} \leq \frac{2}{x+1} \leq \sqrt{\frac{1}{\alpha}} \leq \sqrt{\frac{1}{x}} \leq 1 \leq \frac{2\alpha}{\alpha+1} \leq \sqrt{x} \leq \frac{2y}{y+1} \leq \sqrt{\alpha} \leq \frac{x+1}{2} \leq \sqrt{y}.$$

Fig. 1. Shows the detailed arrangement about members of this inequation.

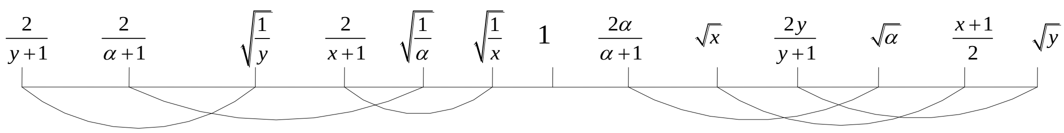

Fig. 1. Geometrical description of the inequation

3 Conclusion and Future Work

This paper solves the problem that came across during the study of distribution of integers in T_3 tree, testifies a series of inequations and provides process of proofs for it. Solutions of this paper is useful to compare inequation and investigate the integer's location in T_3 tree. In the end, there is one amusing problem with the solved problem above, that is changing the condition (1) into the following one

$$1<(\frac{u+1}{2})^2 \le 2\sqrt{\alpha}-1 \le x \le (\frac{2y}{y+1})^2 \le \alpha \le y \le (\frac{\alpha+1}{2})^2 \le w < 4\,.$$

Yields a more complicated distribution about u,x,y,α and w . Furthermore, it has been no what the distribution is if the condition is changed to be

$$1<(\frac{u+1}{2})^2 \le 2\sqrt{\alpha}-1 \le x \le \beta \le (\frac{2y}{y+1})^2 \le \chi \le \alpha \le y \le (\frac{\alpha+1}{2})^2 \le w < 4\,.$$

Hope readers to join and to solve the problems.

Disclaimer

The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

Acknowledgements

The research work is supported by the Ministry of Industry and Information Technology under project on information protection for numerical control systems with No. TC2000H03A and Department of Guangdong Science and Technology under project 2020B1111410002. The authors sincerely present thanks to them all.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Wang X, Jin Y. More Connections on Valuated Binary Tree and Their Applications in Factoring Odd Integers. Asian Research Journal of Mathematics. 2021;17(6):13-26.
DOI: 10.9734/ARJOM/2021/v17i530297.

[2] Wang X, Luo J. Connections on Valuated Binary Tree and Their Applications in Factoring Odd Integers. Asian Research Journal of Mathematics. 2021;17(3):134-153.
DOI: 10.9734/ARJOM/2021/v17i630305.

[3] Wang X, Guo H. Distribution of RSA Number's Divisor on T3 Tree. International Journal of Information and Electronics Engineering. 2019;9(1):23-29.
DOI: 10.18178/IJIEE.2019.9.1.699.

[4] Wang X, Guo H. Some Divisibility Traits on Valuated Binary Trees. International Journal of Information and Electronics Engineering. 2019;9(1):1-11.
DOI:10.17706/ijapm.2019.9.1.1-11.

[5] Wang X. Valuated Binary Tree: A New Approach in Study of Integers. International Journal of Scientific and Innovative Mathematical Research; 2016.
DOI:10.20431/2347-3142.0403008.

[6] Wang X. More on Symmetric Brothers of a Node in a Perfect Binary Tree. Asian Research Journal of Mathematics. 2021;17(6):13-17.

[7] Kuang J. Applied Inequalities. 4th ed. Shandong Science and technology Press; 2010.

[8] Pham KH. Secrets in Inequalities. Harbin Institute of Technology Press. 2010;I.

[9] Pachpatte BG. Mathematical inequalities, North-Holland mathematical library; 2005.

[10] Cerone P, Dragomir SS. Mathematical inequalities. CRC Press; 2011.

[11] Vasile Cirtoaje. Mathematical inequalities. Other methods of creating inequalities and solving inequalities. Harbin Institute of Technology Press. 2021;5.

[12] Han MX, Yan JH. Bound Estimation of Some Special Functions. Asian Research Journal of Mathematics. 2021;17(10): 59-68.
DOI: 10.9734/ARJOM/2021/v17i1030337.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/78399