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Analysis of Stresses and Deflection of Sun Gear by Theoretical and ANSYS Method

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Abstract

Gearing is one of the most critical components in mechanical power transmission systems. This article examines the various stresses and deflection developed in sun gear tooth of planetary gearbox which is used in Grabbing Crane. Article includes checking sun gear wear stresses and bending stresses using IS 4460 equations. Also calculate various forces acting on gear tooth. In this study, perform the calculation for sun gear tooth to calculate bending, shear, wear & deflection using theoretical method. 3D model is created of circular root fillet & trochoidal root fillet of gear tooth for simulation using ProE Wildfire 3. In Pro-E, the geometry is saved as a file and then it is transferred from Pro-E to ANSYS 10 in IGES format. The results of the 3 D analyses from ANSYS are compared with the theoretical values. Comparison of ANSYS results in circular root fillet & trochoidal root fillet also carry out.

Keywords: Bending Stress, Circular Root Fillet, Deflection, Grabbing Crane, Planetary Gearbox, Shear Stress, Sun Gear, Trochoidal Root Fillet, Wear

1. Introduction

In spite of the number of investigations devoted to gear research and analysis there still remains to be developed, a general numerical approach capable of predicting the effects of variations in gear geometry, shear, wear and bending stresses. The objective of this work is to use ANSYS to develop theoretical models of the behavior of planetary gears in mesh, to help to predict the effect of gear tooth stresses and deflection. The main focus of the current research as developed here is to develop and to determine appropriate models of contact elements, to calculate various stresses and using ANSYS and compare the results with theoretical.

The project work mainly deals with

1) Checking of wear stresses & bending stresses using IS 4460 equations for sun gear.

2) Force calculations for planetary gear

3) Calculate the values for sun gear tooth for bending, shear, wear & deflection using theoretical method.

4) Generation of gear tooth profile in Pro-E3.

5) Create 3D model of circular root fillet & trochoidal root fillet of gear tooth for simulation using Pro-E3.

6) Importing Pro-E model in ANSYS in IGES format.

7) Comparison of the results of the 3D analyses from ANSYS with the theoretical values.

8) Comparison of ANSYS results in circular root fillet & trochoidal root fillet.

Shanmugasundaram Sankar, Maasanamuthu Sundar Raj & Muthusamy Nataraj [1] have introduced Corrective measures are taken to avoid tooth damage by introducing profile modification in root fillet. Tesfahunegn and Rosa [2] investigated the influence of the shape of profile modifications on transmission error, root stress and contact pressure through non linear finite element approach. Chun-Fang Tsai ,Tsang-Lang Liang & Shyue-Cheng Yang [3] prepared a complete mathematical model of the planetary gear mechanism with double circular-arc teeth is developed. Ravichandra Patchigolla and Yesh P. Singh [4] developed a program using ANSYS Parametric De- sign Language (APDL) to generate 1, 3 or 5 tooth segment finite element models of a large spur gear. Faydor L. Litvin, Alfonso Fuentes, Daniele Vecchiato, and Ignacio Gonzalez-Perez [5] proposed a new types of planetary and planetary face-gear drives & the new designs are based on regulating backlash between the gears and modifying the tooth surfaces to improve the design.

2. Terminology

2.1. Terminology—Spur Gears

Refer: **Figure 1**.

Diametral pitch (d_p): The number of teeth per one inch of pitch circle diameter.

Module (m): The length, in mm, of the pitch circle diameter per tooth.

Circular pitch (p): The distance between adjacent teeth measured along the pitch circle diameter

Addendum (h_a): The height of the tooth above the pitch circle diameter.

Centre distance (a): The distance between the axis of two gears in mesh.

Circular tooth thickness (Ctt): The width of a tooth measured along the are at the pitch circle diameter.

Dedendum (h_f): The depth of the tooth below the pitch circle diameter.

Outside diameter (D_o): The outside diameter of the gear.

Base Circle diameter (D_b): The diameter on which the involute teeth profile is based.

Pitch circle dia (D): The diameter of the pitch circle.

Pitch point: The point at which the pitch circle diameters of two gears in mesh coincide.

Pitch to back: The distance on a rack between the pitch circle diameter line & the rear face of the rack.

Pressure angle: The angle between the tooth profile at the pitch circle diameter & a radial line passing through the same point.

Whole depth: The total depth of the space between adjacent teeth.

2.2. Terminology—Planetary Gear Train

Refer **Figures 2** and **3**.

Sun: The central gear

Planet Gear: Peripheral gears, of the same size, meshed with the sun gear and Annulus

Planet carrier: Holds one or more peripheral planet gears, of the same size, meshed with the sun gear

Ring Gear/Annulus: An outer ring with inward-facing teeth that mesh with the planet gear or gears

No. of teeth on Gear (Z)

No. of teeth on Sun Gear (S)

No. of teeth on Planet Gear (P)

No. of teeth on Pinion Gear 1 (P1)

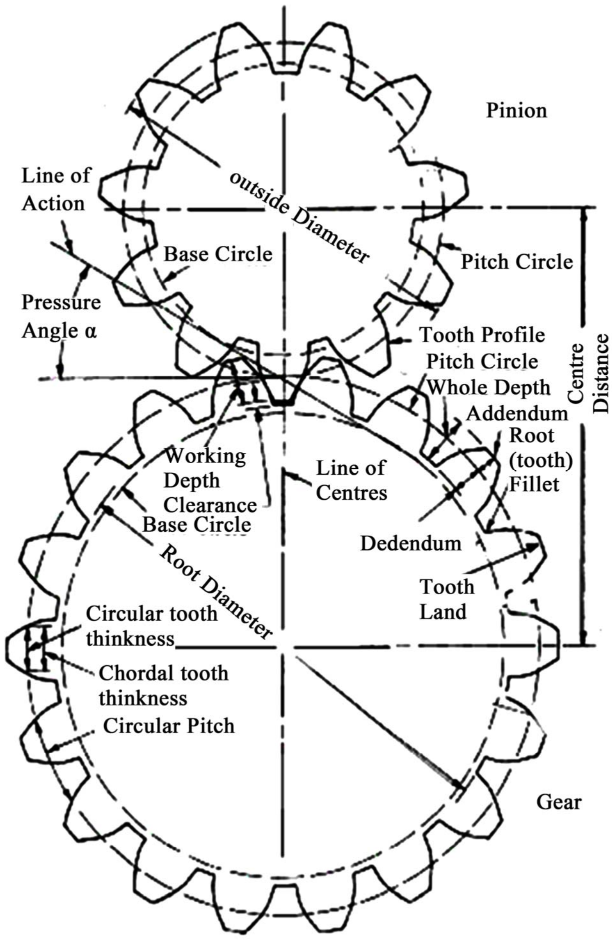


Figure 1. Terminology—spur gears.

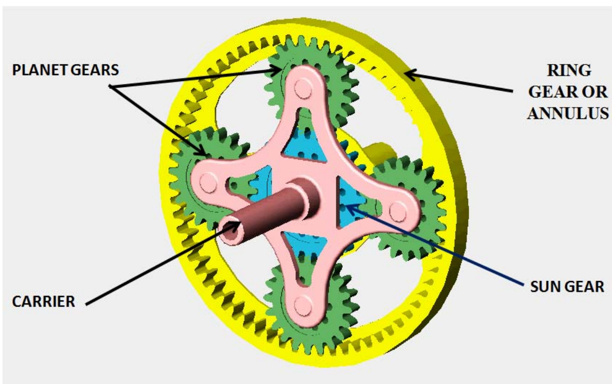


Figure 2. Planetary gear train.

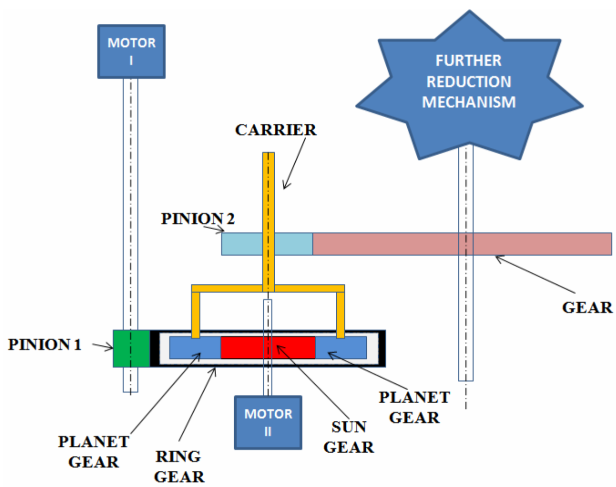


Figure 3. Schematic diagram of planetary gear train.

No. of teeth on Pinion Gear 2 (P2)
No. of teeth on Ring Gear External (RE)
No. of teeth on Ring Gear Internal (RI)
Total Gear Reduction Ratio (TR)

How Planetary Gear Train works? For different conditions of Motor I & II *in* **Figure 3**, *Gear ratios are shown in* **Table 1**.

3. Grabbing Crane Specification

For Photograph Refer: **Figure 4**

Capacity: 10 Ton wire rope (weight of material handled + grab)

Duty: Class 4/12 Hr

Location: Outdoor

Table 1. Gear ratio.

Motor's On & Off Condition	Motor "I" On & Motor "II" Off	Motor "I" Off & Motor "II" On	Motor "I" On & Motor "II" On (Opposite Direction)
1st Stage	RE/P1	----	
2nd Stage	(RI + S)/RI	(RI + S)/S	$TR = \frac{TR1 \times TR2}{TR1 + TR2}$
3rd Stage	G/P2	G/P2	
Total Reduction	TR1 = (RE/P1) × ((RI+S)/RI) × (G/P2)	TR2= ((RI + S)/S) × (G/P2)	

Figure 4. Grabbing crane.

Total weight of trolley: 13 T
Total weight of crane: 71 T
Ambient temperature: 50˚C
Lubrication: Group
Operations from: Closed cabin
Grab bucket capacity: 3.5 m^3
Material handled: Blast furnace slag
Bulk density: 1.1 - 1.2 T/m^3
Weight of material handled: 4 T
Dead weight of grab bucket: 5.5 T
Different Parts in figure 5 mentioned in Table2

4. Construction of Gearbox

There are two basic operations involved in this gearbox, which are

1) Holding
2) Opening-closing.

4.1. Holding

It is a 3-stage mechanism out of which, first two stages are helical & third stage which is output of this mechanism is a spur gear pair.

We have used spur gear in output stage because,

1) Due to large P.C.D. of exterior teeth of annulus, it is very difficult to machine helical gearing on it, hence spur teeth are machined.

2) The output shaft of the holding drum is heavily loaded. As per the requirement of high hoist speed, the rotational speeds of the shafts are very high, which leads to selection of large capacity of bearings.

If helical gearing is used in the output of this gearbox, additional thrust factors are generated which further increase the bearing size. Hence, spur is preferred in the output stage of the gearboxes of such applications.

4.2. Opening-Closing

It is also a 3-stage mechanism in which, first stage is spur & rest two are helical. Helical gears are used to achieve desired reduction & maximum efficiency. Output of this mechanism is given to SUN of "Planetary Gear System"

4.3. Planetary System

Planetary system consists of one SUN gear, three PLANETS & an ANNULUS. When holding motor is put on, SUN is kept fixed & when opening-closing motor is in operation; annulus is kept fixed.

5. Working of Gear Box

The grab bucket is closed as follows (**Figure 6(b)**). The

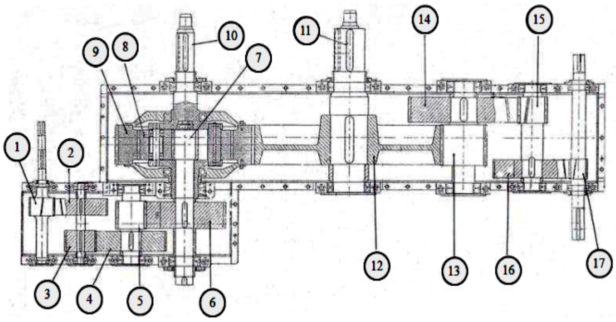
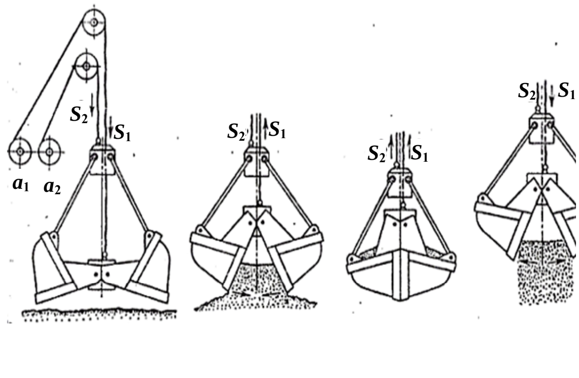


Figure 5. Planetary gears system used in grab crane's hoist.



(a) (b) (c) (d)

Figure 6. Principal of operation of a double rope grab bucket.

Table 2. Part list (Refer Figure 5).

Part No.	Description	Material
1	Input Helical Pinion Z = 31 m = 4 (RH)	EN24
2	Helical Gear Z = 68m = 4 (LH)	EN9
3	Spur Pinion Z = 35 m = 4	EN24
4	Spur Gear Z = 75 m = 4	EN9
5	2nd Spur Pinion Z = 23 m = 6	EN24
6	Spur Gear Z = 67 m = 6	EN9
7	Sun Pinion (Spur) Z = 18 m = 6	17CrNiMo6
8	Planet (Spur) Z=26 m = 6	17CrNiMo6
9	Annulus Internal Z = 72 m = 6, External Z = 76 m = 9	EN9
10	Opening Closing Drum Shaft	EN19
11	Holding Drum Shaft	EN19
12	Spur Gear Z = 104 m = 9	EN9
13	Spur Pinion Z = 24 m = 9	EN24
14	Helical Gear Z = 79 m = 7 (RH)	EN9
15	Helical Pinion Z = 20 m = 7 (LH)	EN24
16	Helical Gear Z = 80 m = 5 (LH)	EN9
17	Input Helical Pinion Z = 19 m = 5 (RH)	EN24

closing drum a_1 rotates to lifting, *i.e.*, counterclockwise while the hoisting drum a_2 is immobile.

The closing rope s_1 is tightened, the movable cross-member goes upwards & the scoops cut into material as they are gradually brought together until their edges are tightly compressed.

In raising (**Figure 6(c)**), both drums rotate clockwise.

To dump the grab bucket(**Figure 6(d)**), the hoisting drum is braked & closing drum revolves for descent (clockwise); this causes the bucket scoops to open under the action of their own weight & that of the material & the contents are dumped.

This cycle continues through the duty hours.

5.1. Normal Rating

The normal ratings of the gears is the allowable continuous load for 12 hours running time per day.

5.2. Duty Factor: (as per IS 4137)

1) For Class III Crane: Duty factor for wear = 0.6 Duty factor for strength = 1.4
2) For Class IV Crane: Duty factor for wear = 0.7 Duty factor for strength = 1.6

6. Checking Sun Gear Wear Stresses & Bending Stresses Using IS 4460 Equations

Capacity = 10 tones. Speed = 20 m/min. From IS standards,

$$\text{KW ratings} = \frac{\text{Capacity} \times \text{Speed}}{6.12 \times \text{efficiency}}$$

But Efficiency = $(0.95)^n \times (0.99)^m$ where n = number of stages in gearbox = 9. m = number of rotating sheaves between the rope drum & equalizer = 5

Efficiency = $(0.95)^9 \times (0.99)^5 = 0.5994$

$$\text{KW rating} = \frac{10 \times 20}{6.12 \times 0.5994} = 54.524$$

Considering 60% of ratings, KW rating = 0.6 × 54.524 = 32.71

Further HP required = 32.71/0.735= 44.47 Assuming HP rating = 45

6.1. Calculations for Planetary System Speeds

Refer: **Figure 7**

Let,

T_S = number of teeth on sun

T_P = number of teeth on planet

T_A = internal number of teeth on annulus

N_A = speed of annulus
N_S = speed of sun
N_P = speed of planet
N_C = speed of carrier

Different possible conditions for planetary gear drive system are mentioned in **Table 3** and speed calculations for planetary unit are mentioned in **Table 4**

Calculations for opening closing mechanism is mentioned in **Table 5**

For mentioned gearbox, Speed calculations for planetary unit are mentioned in **Table 6**

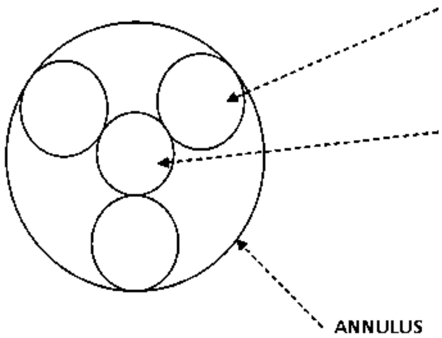


Figure 7. Schematic diagram of planetary gear train.

From the above table,
Speed of annulus N_A is given as

$$N_A = N_C\left(\frac{T_A + T_S}{T_A}\right) - \frac{N_S T_S}{T_A}$$

$$N_C = \frac{N_A T_A + N_S T_S}{T_A + T_S}$$

Table 3. Gear drive system design.

I/P	Sun	Carrier	Ring	Sun, ring	Carrier, sun	Ring carrier
O/P	Carrier ring	Sun, Ring	Carrier, sun	Carrier	Ring	Sun

Table 4. Speed calculation for planetary unit.

Sun	Planets	Carrier	Annulus
+1	+1	+1	+1
–1	$\frac{T_S}{T_P}$	0	$\frac{T_S}{T_P} \times \frac{T_P}{T_A} = \frac{T_S}{T_A}$
0	$1 + \frac{T_S}{T_P}$	1	$1 + \frac{Ts}{T_A}$

Table 5. Calculations for opening closing mechanism.

H.P Ratings of Gears							
Eq. Running Time: 12.00 Hrs/Day							
Duty Class: 4							
Input R.P.M = 720				H.P Ratings: 45			
Particulars		**1st Stage**		**2nd Stage**		**3rd Stage**	
		Pinion	Gear	Pinion	Gear	Pinion	Gear
Material		EN24	EN9	EN24	EN9	EN24	EN9
Module:	m	4		4		6	
No. of Teeth:	Z	31	68	35	75	23	67
R.P.M:	N	720	328.23	328.23	153.18	153.18	52.58
Face Width (mm):	B	80		80		120	
COS of Helix Angle		0.99		1.00		1.00	
Speed Factor For Wear: $_c$	X	0.282	0.332	0.332	0.372	0.372	0.461
Zone Factor:	Y_Z	2.762		4.00		2.90	
Surface Stress(KG/mm^2):	σ_c	3.87	2.11	3.87	2.11	3.87	2.11
Pitch Factor:	K_1	5.79		5.79		8.001	
Duty Factor(Wear):	SF_c			0.7			
Speed Factor For Strength:	X_b	0.285	0.315	0.315	0.39	0.39	0.466
Strength Factor × Def Fac:	Y	0.96	0.869	0.775	0.76	0.721	0.625
Bending Stress (Kg/mm^2):	σ_b	37.3	21.5	37.3	21.5	37.3	21.5
Duty Factor(Strength):	SF_b			1.6			
A = MZNB × 69.8 × 10^{-8}/Eff.		5.865	5.865	3.019	3.019	2.083	2.083
$L_C = X_C Y_Z \sigma_C K_1/SF_c$		24.93	16.00	42.51	25.97	47.72	32.24
$L_B = X_b Y \sigma_b M / SF_b$		25.51	14.71	22.7664	15.93	39.32	23.48
Wear HP = A × LC		146.23	93.84	128.34	78.4	99.4	67.15
Strength HP = A × Lb		149.62	86.27	68.724	48.09	81.92	48.91

Table 6. Speed calculation for planetary unit.

	Sun	Planets	Carrier	Annulus
Carrier Fixed, Sun Rotated By -N_s	N_S	$-N_S\dfrac{T_S}{T_P}$	0	$-N_S\dfrac{T_S}{T_A}$
Sun Fixed, Carrier Rotated By -N_c	0	$N_C\times\left(1+\dfrac{T_S}{T_P}\right)$	N_C	$N_C\times\left(1+\dfrac{T_S}{T_A}\right)$
Total Motion	N_S	$N_C\left(\dfrac{T_P+T_S}{T_P}\right)-\dfrac{N_ST_S}{T_P}$	N_C	$N_C\left(\dfrac{T_A+T_S}{T_A}\right)-\dfrac{N_ST_S}{T_A}$

When annulus is fixed, $N_A = 0$

$$N_C=\frac{N_ST_S}{T_A+T_s}=\frac{52.79\times 18}{72+18}=10.52\text{ rpm}$$

Planet speed is given as,

$$N_P=N_C\left(\frac{T_P+T_S}{T_P}\right)-\frac{N_ST_S}{T_P}$$

$$N_P=10.52\left(\frac{26+18}{26}\right)-\frac{52.58\times 18}{26}$$

$$N_P=\ 18.68\text{ rpm}$$

Calculations of HP for planetary system is mentioned in **Table7**

From the above table, speed of annulus N_A is given as

$$N_A=N_C\left(\frac{T_A+T_S}{T_A}\right)-\frac{N_ST_S}{T_A}$$

$$N_C=\frac{N_AT_A+N_ST_S}{T_A+T_S}$$

when annulus is fixed, $N_A = 0$

$$N_C=\frac{N_ST_S}{T_A+T_s}=\frac{52.79\times 18}{72+18}=10.52\text{ rpm}$$

Planet speed is given as,

$$N_P=N_C\left(\frac{T_P+T_S}{T_P}\right)-\frac{N_ST_S}{T_P}$$

$$N_P=10.52\left(\frac{26+18}{26}\right)-\frac{52.58\times 18}{26}$$

$$N_P=18.68\text{ rpm}$$

But for planetary system

Strength of sun $= 3\times \text{HP}_{\text{normal}} = 3\times 14.32 = 42.06\,\text{HP}$

Strength of sun in wear $= \text{HP}_{\text{normal}}/3 = 10.52/3 = 3.5\,\text{HP}$

Table 7. Calculation of HP for planetary system.

H.P Ratings of Gears			
Eq. Running Time : 12.00 Hrs/Day			
Duty Class : 4			
Input R.P.M = 720			
Particulars		Pinion	Gear
Material		EN24	EN24
Module:	m	6	6
No. of Teeth:	Z	18	26
R.P.M:	N	52.58	18.68
Face Width (mm):	B	120	120
COS of Helix Angle		1.00	1.00
Speed Factor For Wear:	X_c	0.47	0.57
Zone Factor:	Y_Z	1.65	1.65
Surface Stress(KG/mm^2):	σ_c	2.11	2.11
Pitch Factor:	K_1	8.008	8.008
Duty Factor(Wear):	SF_c	0.7	0.7
Speed Factor For Strength:	X_b	0.45	0.52
Strength Factor × Def Fac	Y	0.64	0.615
Bending Stress (Kg/mm^2):	σ_b	23.6	23.6
Duty Factor(Strength):	SF_b	1.6	1.6
A = MZNB × 69.8 × 10^{-8}/Eff.		0.562	0.287
$L_C = X_C\, Y_Z\, \sigma_C\, K_1/SF_c$		18.719	22.702
$L_B = X_b\, Y\, \sigma_b M/SF_b$		25.488	28.3023
Wear HP = A × LC		10.52	6.515
Strength HP = A × Lb		14.32	8.122

	Pinion	Gear
Wear HP	3.52	6.515
Strength HP	42.06	8.122

Hence HP rating for planetary system will be minimum of the above 4 HP values

$(\text{HP})_{\text{Planetary}} = 3.52$

6.2. Digging Force Calculations

Normal reaction
= Capacity
= Volume of bucket × density of material
= 3.5 × 1.1 × 1000
= 3850 Kg
= 3.85 T

Assuming coefficient of frictions, μ = 0.1
Friction force = μN = 0.1 × 3.85= 0.385 T
Torque = Force × Perpendicular distance
= 0.385 × 1.8
= 0.693 Tm=693 kg-m
Time required to grab 1 Ton of slag
= Displacement/Velocity
= 1.8/20 = 0.09 min°
Speed = 0.5/0.09 = 5.56 rpm
From standard formulae,

$$T = \frac{736 \times \mathrm{HP}}{\mathrm{rpm}}$$

$$693 = \frac{736 \times \mathrm{HP}}{5.56}$$

$HP = HP_{induced} = 5.23$

Since, $HP_{induced} > HP_{Planetary}$

Design unsafe

Modified calculations for HP ratings for planetary gear system is mentioned in **Table 8**.

But for planetary system
Strength of sun = 3 × HP_{normal}
= 3 × 17.06
= 51.18 HP
Strength of sun in wear = $HP_{normal}/3$
= 35.748/3
= 11.916 HP

	Pinion	Gear
Wear HP	11.916	24.3
Strength HP	51.18	11.37

Hence HP rating for planetary system will be minimum of the above 4 HP values
$(HP)_{Planetary} = 11.37$

6.3. Digging Force Calculations

Normal reaction
= Capacity
= Volume of bucket × density of material
= 3.5 × 1.1 × 1000
= 3850 Kg
= 3.85 T

Assuming coefficient of frictions, μ = 0.1

Table 8. Modified calculations. using pinion [3% nickel steel with BHN 620 (case)] & gear [5% nickel steel with BHN 600(case)].

H.P Ratings of Gears			
Eq. Running Time : 12.00 Hrs/Day			
Duty Class : 4			
Input R.P.M = 720			
Particulars		1st Stage	
		Pinion	Gear
Material		3% Nickel Steel	5% Nickel Steel
Module	m	6	6
No. of Teeth:	Z	18	26
R.P.M:	N	52.58	18.68
Face Width (mm):	B	120	120
COS of Helix Angle		1.00	1.00
Speed Factor For Wear:	X_c	0.47	0.57
Zone Factor:	Y_Z	1.65	1.65
Surface Stress(KG/mm^2):	σ_c	7.17	7.87
Pitch Factor:	K_1	8.008	8.008
Duty Factor(Wear):	SF_c	0.7	0.7
Speed Factor For Strength:	X_b	0.45	0.52
Strength Factor × Def Fac	Y	0.64	0.615
Bending Stress (Kg/mm^2):	σ_b	28.12	33.07
Duty Factor(Strength):	SF_b	1.6	1.6
A = MZNB × 69.8 × 10^{-8}/Eff.		0.562	0.287
$L_C = X_C Y_Z \sigma_C K_1/SF_c$		63.61	84.67
$L_B = X_b Y \sigma_b M / SF_b$		30.36	39.65
Wear HP = A × LC		35.748	24.3
Strength HP = A × Lb		17.06	11.37

Friction force = μN = 0.1 × 3.85= 0.385 T
Torque = Force × Perpendicular distance
= 0.385 × 1.8
= 0.693 Tm=693kg-m
Time required to grab 1 Ton of slag
= Displacement / Velocity
= 1.8/20 = 0.09 min
Speed = 0.5/0.09 = 5.56 rpm
From standard formulae,

$$T = \frac{736 \times HP}{\mathrm{rpm}}$$

$$693 = \frac{736 \times HP}{5.56}$$

$HP = HP_{induced} = 5.23$
Since, $HP_{induced} < HP_{Planetary}$

Design safe

Table 9 contains force calculations for mentioned grabbing crane hoist gear box.

Table 9. Gear forces calculation.

Particulars		Module	No of Teeth	Power (P)			N	Torque /Moment	PCD	R	Ft	COSα	SINα	Fn	Fr
Unit		mm	No.	HP	Factor	KW	RPM	N-mm	mm	mm	N	COS 20	SIN 20	N	N
		Formulae						(P×60×1000×1000) /(2×(22/7) ×N)	PCD/2		Ft = T/R			Ft/COSα	Fn × SINα
1st Stage	Pinion	4	31	45	0.746	33.57	720	445056.8182	124	62	7178.3358	0.93969	0.34202	7639.025	2612.701
	Gear		68	45	0.746	33.57	328.23	976269.412	272	136	7178.4516	0.93969	0.34202	7639.149	2612.743
2nd Stage	Pinion	4	35	45	0.746	33.57	328.23	976269.412	140	70	13946.706	0.93969	0.34202	14841.77	5076.186
	Gear		75	45	0.746	33.57	153.18	2091923.94	300	150	13946.16	0.93969	0.34202	14841.19	5075.987
3rd Stage	Pinion	6	23	45	0.746	33.57	153.18	2091923.94	138	69	30317.738	0.93969	0.34202	32263.46	11034.75
	Gear		67	45	0.746	33.57	52.58	6094349.735	402	201	30320.148	0.93969	0.34202	32266.03	11035.63
4th Stage (Planetary)	Sun	6	18	45	0.746	33.57	52.58	6094349.735	108	54	112858.33	0.93969	0.34202	120101.3	41077.07
	Planetary (3nos)	6	26	45	0.746	33.57	18.68	17154224.26	156	78	219925.95	0.93969	0.34202	234040.3	80046.5
	Planetary (1 No)	6	26								73308.651			78013.44	26682.17

7. Theoretical Stress & Deflection Calculation

7.1. Bending Stress

σ_b = Bending Stress

The classic method of estimating the bending stresses in a gear tooth is the Lewis equation. It models a gear tooth taking the full load at its tip as a simple cantilever beam. Refer: **Figure 8**

Lewis Bending Stress From,

$$\sigma_b = \frac{Wt\ Pd}{FY} = \frac{Wt\ \pi}{mFY}$$

where:

Wt is the tangential load

Pd is the diametral pitch

F is the face width

Y is the Lewis form factor

m is the module

Since tangential load

Ft = Wt=112858.33 act on 3 no. teeth,

hence tangential load

Ft = Wt act on single teeth = 112858.33 /3 = 37619.443 N

Wt = 37619.443 N

F is the face width = 120 mm

Y is the Lewis form factor for 18 No. Teeth = 0.308

m is module = 6 mm

σ_b Bending Stress,

$$\sigma_b = \frac{Wt\ \pi}{mFY}$$

$$\sigma_b = \frac{37619.443 \times \pi}{6 \times 120 \times 0.308}$$

$$\sigma_b = 532.94\,\mathrm{N/mm^2}$$

7.2. Shear Stress

σ_s = Shear Stress

$$\sigma_s = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{Ft}{A}$$

$$\sigma_s = \frac{Ft}{\text{b} \times \text{Tooth Width}}$$

$$\sigma_s = \frac{Ft}{b \times (\pi \times \mathrm{m})/2}$$

Since tangential load

Ft = Wt=112858.33 act on 3 no. teeth,

hence tangential load

Ft = Wt act on single teeth

= 112858.33 /3 = 37619.443 N

$$\sigma_s = \frac{Ft}{b \times (\pi \times \mathrm{m})/2}$$

$$\sigma_s = \frac{37619.443}{120 \times (\pi \times 6)/2}$$

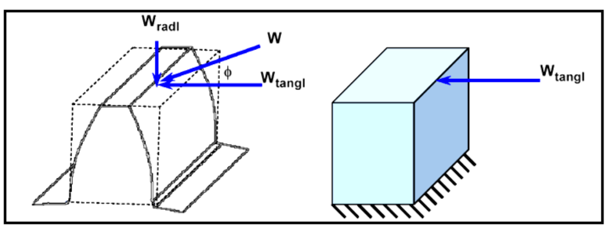


Figure 8. Tooth forces in spur gear.

$$\sigma_s = 33.2495077\ \mathrm{N/mm^2}$$

7.3. Wear Stress

σ_c = *Wear Stress*

$$\sigma_c = 0.74\frac{i+1}{a}\sqrt{\frac{i+1}{ib}}\sqrt{E[M_t]}$$

a = *Centre to centre distance* = 132 mm
i = *Gear Ratio* = 2.81
E = *Modulus of Elasticity* = 2×10^5 N /mm^2

$$\sigma_c = 0.74\frac{2.81+1}{132}\sqrt{\frac{2.81+1}{2.81\times 120}}\sqrt{2\times 10^5[6094349.735_t]}$$

$$\sigma_c = 2509.154561\ \mathrm{N/mm^2}$$

7.4. Deflection

Deflection = A/B
Where,
A = Wt × L^3
Wt = Tangential load
Wt = 37619.443 N
L = H = Tooth Height
= 2.25 × Module
= 2.25 × 6
= 13.5 mm
Therefore,
A =37619.443 × 13.5^3
A = 92557936.61
&, B= 3 × E × I
E = Modulus of Elasticity
E = 2×10^5 N /mm^2
I = Moment of Inertia
I = (a × b^3)/12
A = Face Width
a = 120 mm
b = Tooth Width
b = (π/4) × Module
b= (π/4) × 6
b = 4.714 mm
Hence,
I = 120 × 4.714^3
I = 4190.90379 mm^4
Therefore,
B = 3 × 2 × 10^5 × 4190.90379
B = 2514542274
Therefore,
Deflection = 92557936.61/2514542274
Deflection = 0.03680906 mm

8. Geometrical Modeling

Consider the involute spur Gear, gear tooth of circular fillet illustrated in **Figure 9** where point O is the center of the gear, axis Oy is the axis of symmetry of the tooth & point B is the point where the involute profile starts (form the form circle rs). A is the point of tangency of the circular fillet with the root circle rf. Point D lying on (e2) OA represents the center of the circular fillet. Line (e3) is tangent to the root circle at A & intersects with line (e1) at C. The fillet is tangent to the line (e1) at point E. Since it is always rs > rf , the proposed circular fillet can be implemented without exceptions on all spur gears irrelevant of number of teeth or other manufacturing parameters. A comparison of the geometrical shape of a tooth of circular fillet with that of standard (trochoidal) fillet is presented in **Figures 10** and **11**.

9. Element Analysis

A finite element model with a single tooth is considered for analysis. Gear material strength is a major consideration for the operational loading & environment. In modern practice, the heat treated alloy steels are used to overcome the wear resistance. ANSYS version 10.0 software is used for analysis. In this work, heat treated alloy is taken for analysis. The gear tooth is meshed in 3-dimensional (3-D) solid 16 nodes 92 elements with fine mesh. SOLID92 has a quadratic displacement behavior & is well suited to model irregular meshes.

For sun gear used in grab crane's hoist **Figure 12** and **13** indicates the PRO-E (3-D View) of Trochodial and Circular Fillet Tooth respectively. Also **Figures 14** and **15** indicates the Meshing of Trochodial and& Circular Fillet Tooth respectively in ANSYS.

The material properties chosen for analysis are as follow

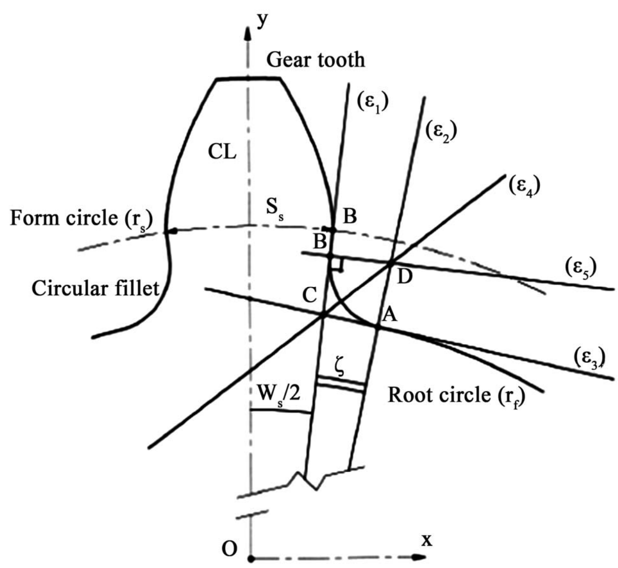


Figure 9. Geometry of the circular fillet.

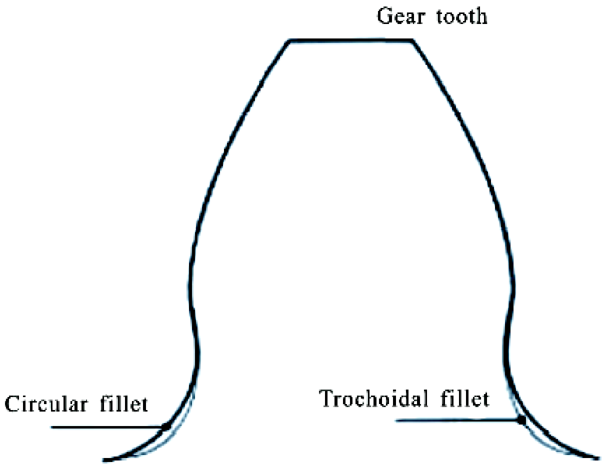


Figure 10. Superposition of circular fillet on a standard tooth.

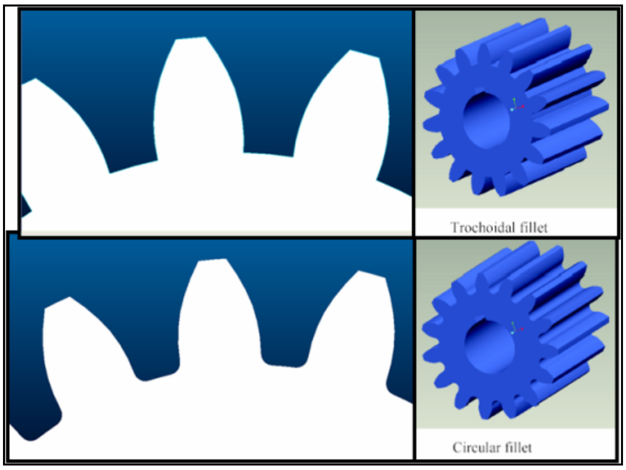


Figure 11. Geometrical modeling: trochoidal & circular fillet spur gear.

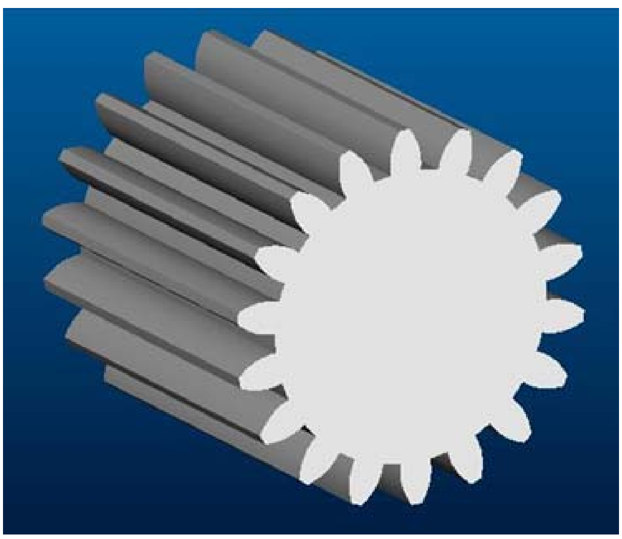


Figure 12. Trochodial fillet tooth sun gear in PRO-E (3D view).

Figure 13. Circular fillet tooth sun gear in PRO-E (3D view).

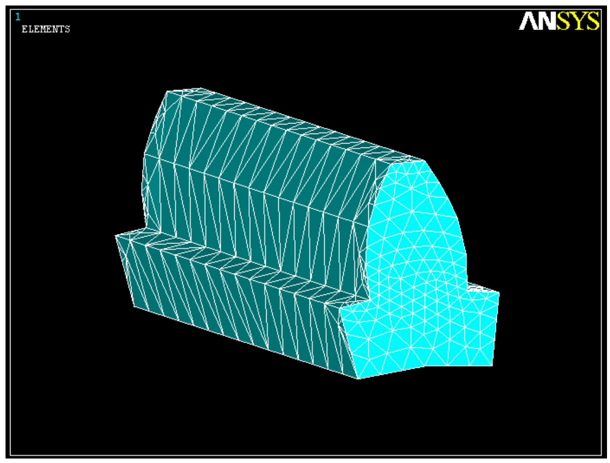


Figure 14. Meshing of trochodial tooth fillet sun gear in ANSYS (3D view).

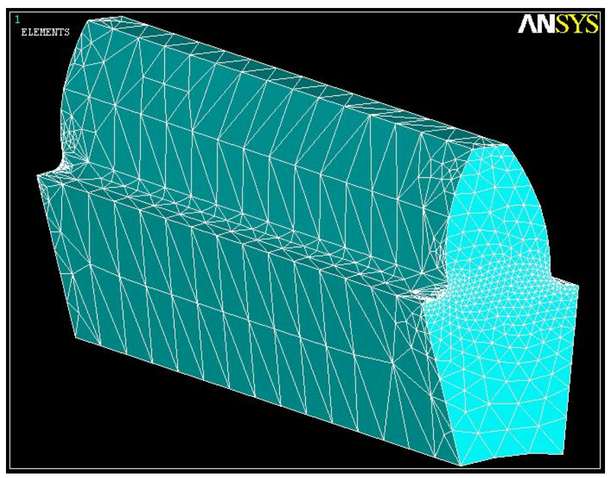


Figure 15. Meshing of circular fillet tooth sun gear in ANSYS (3D view).

Material properties.

Gear material (Sun Gear): [3% nickel steel with BHN 620 (case)]

Density: 7800 kg/m^3

Young's modulus: 2×10^5 N /mm^2

Poisons ratio: 0.3

Yield strength: 28.12 kg/mm^2

10. Results

Refer **Figures 16-23** for ANSYS. Analysis for bending, shear, wear stress & deflection of sun gear used in grab crane's hoist. All results summarized in **Table 10.**

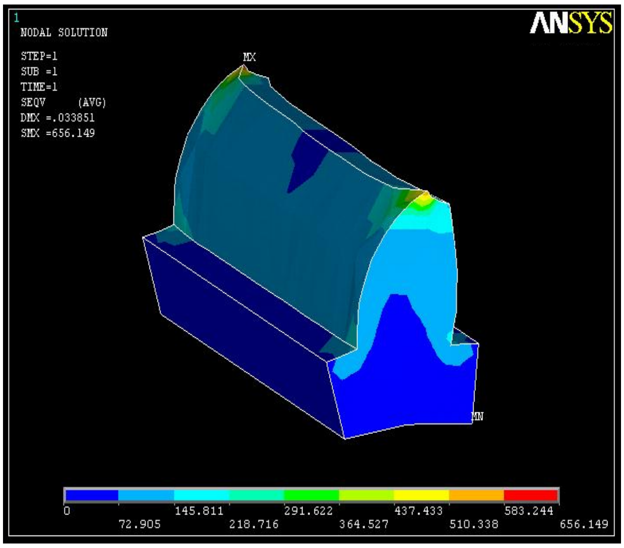


Figure 16. Bending stress trochoidal fillet result: maximum induced σ_b = 656.149 N/mm^2.

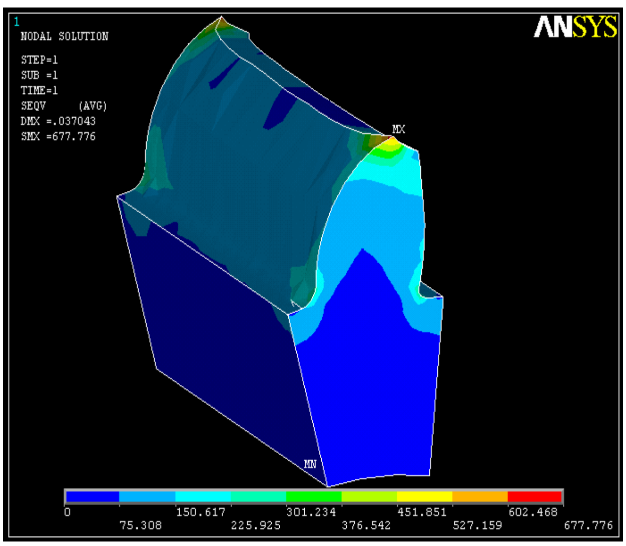


Figure 17. Bending stress circular fillet radius: maximum induced σ_b = 677.776 N/mm^2.

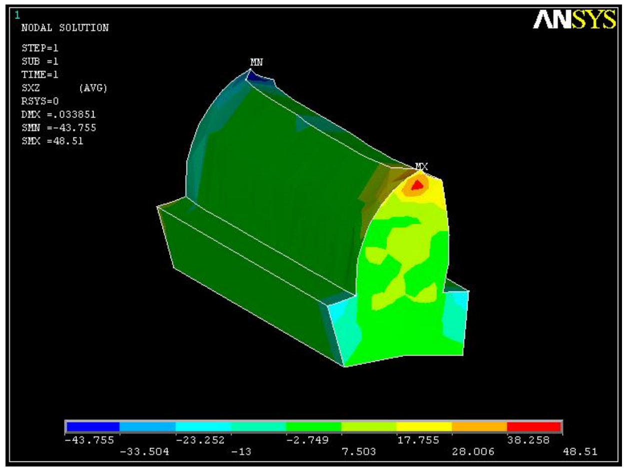


Figure 18. Shear stress-trochoidal fillet result: maximum induced σ_s = 48.51 N/mm^2.

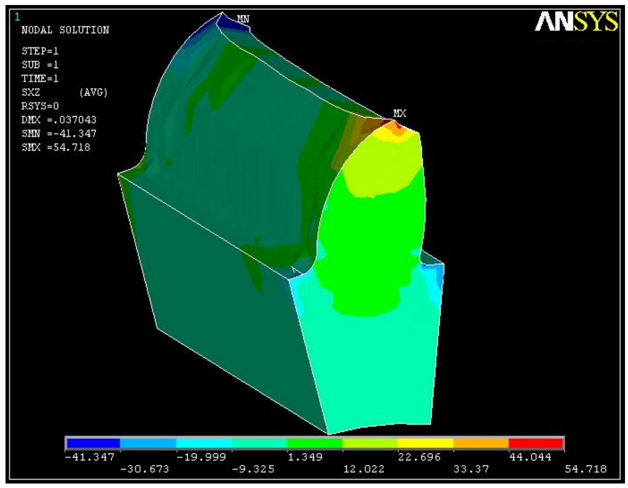


Figure 19. Sher stress circular fillet result: maximum induced σ_s = 54.71 N/mm^2.

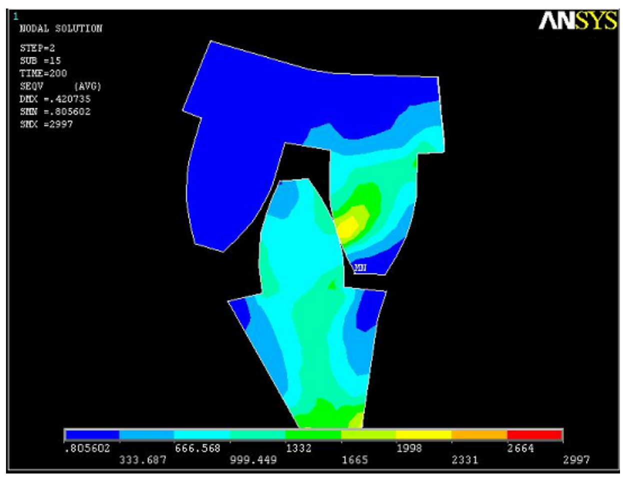


Figure 20. Wear stress trochoidal fillet result:- maximum wear σ_c = 1998 N/mm^2.

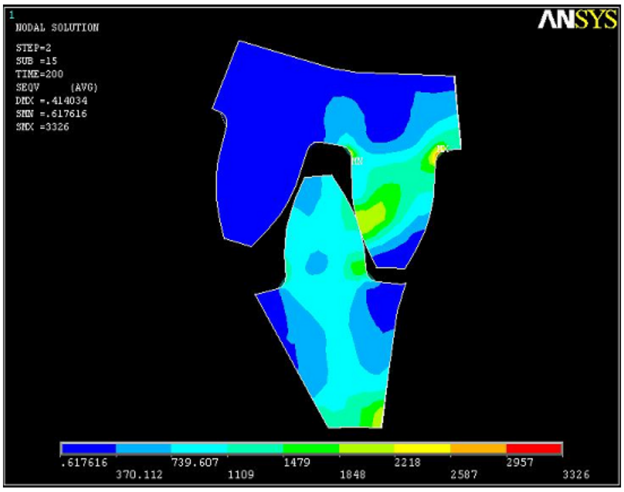


Figure 21. Wear stress circular fillet result: maximum wear σ_c = 1848 N/mm^2.

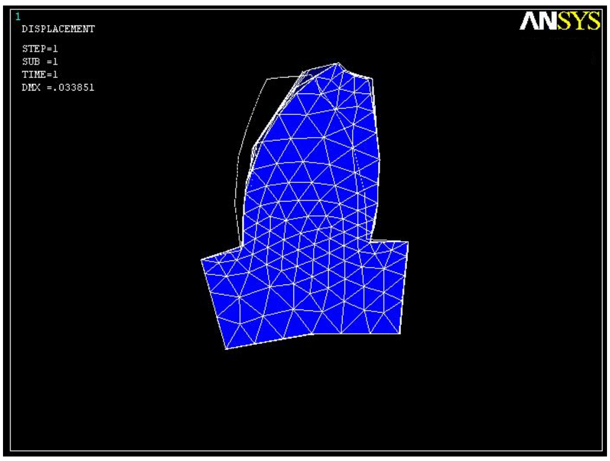


Figure 22. Deflection trochoidal fillet result: deflection = 0.033851 mm.

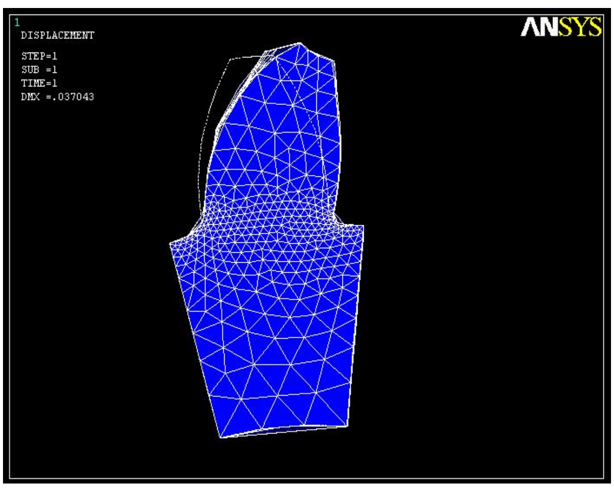


Figure 23. Deflection circular fillet result: deflection = 0.037043 mm.

Table 10. Bending, shear & wear stresses & deflection result.

Analysis	Theoretical Value	BY ANSYS	
		For Trochoidal Fillet Radius	For Circular Fillet Radius
Bending Stress (N/mm^2)	532.94	656.149	677.776
Shear Stress (N/mm^2)	33.24	48.51	54.71
Wear Stress (N/mm^2)	2509.1546	1998	1848
Deflection (mm)	0.0368090	0.033851	0.037043

11. Conclusions

ANSYS results for various stresses & deflection are nearer to theoretical values for sun gear of planetary gear system of grabbing crane. There is appreciable reduction in bending & shear stress value for trochoidal root fillet design in comparison to that of stresses values in circular root fillet design. Also there is increase in wear stress value for trochoidal root fillet design in comparison to that of stresses values in circular root fillet design. The investigation result infers that the deflection in trochoidal root fillet is also less comparing to the circular root fillet gear tooth. However, from the foregoing analysis it is also found that the circular fillet design is more optimum for lesser number of teeth in pinion & trochoidal fillet design is more suitable for higher number of teeth in gear (more than 17 teeth) & whatever may be the pinion speed. In addition to that the ANSYS results indicates that the gears with trochoidal root fillet design will result in better strength, reduced bending stress & also improve the fatigue life of gear material. Further work shall be done to calculate actual theoretical value of trochoidal & circular root fillet gear tooth & then carried out comparison with ANSYS results.

12. References

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