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Uniqueness of Meromorphic Functions Concerning Differential Monomials*

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Abstract

Considering the uniqueness of meromorphic functions concerning differential monomials ,we obtain that, if two non-constant meromorphic functions $f(z)$ and $g(z)$ satisfy $E_k(1, f^n f') = E_k(1, g^n g')$, where k and n are tow positive integers satisfying $k \geq 3$ and $n \geq 11$, then either $f_{(z)} = c_1 e^{cz}, g_{(z)} = c_2 e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1 c_2)^{n+1} c^2 = -1$ or $f = tg$ for a constant t such that $t^{n+1} = 1$.

Keywords: Meromorphic Function, Sharing Value, Uniqueness

1. Introduction and Main Results

In this paper we use the standard notations and terms in the value distribution theory [1].

Let $f(z)$ be a nonconstant meromorophic function on the complex plane C. Define $E(a, f) = \{z | f(z) - a = 0\}$, where a zero point with multiplicity m is counted m times in the set. If these zero points are only counted once, then we denote the set by $\overline{E}(a, f)$. Let k a positive integer. Define
$E_k(a, f) = \{z | f(z) - a = 0, \exists i, 1 \leq i \leq k, st. f^{(i)}(z) \neq 0\}$,
where a zero point whit multiplicity m is counted m times in the set.

Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. If $E(a, f) = E(a, g)$, then we say that $f(z)$ and $g(z)$ share the value **CM**; if $\overline{E}(a, f) = \overline{E}(a, g)$, then we say that $f(z)$ and $g(z)$ share the value **IM**.

Additional, we denote by $N_k(r, f)$ the counting function for poles of $f(z)$ with multiplicity $\leq k$, and by $\overline{N}_{k)}$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r, f)$ be the counting function for poles of $f(z)$ with multiplicity $\geq k$, and by $\overline{N}_{(k}(r, f)$ the corresponding one for which multiplicity is not counted. Set
$N_k(r, f) = \overline{N}(r, f) + \overline{N}_{(2} + \cdots + \overline{N}_{(k}(r, f)$, Similary, we have the notation: $N_{k)}\left(r, \frac{1}{f}\right)$, $\overline{N}_{k)}\left(r, \frac{1}{f}\right)$, $N_{(k}\left(r, \frac{1}{f}\right)$, $\overline{N}_{(k}\left(r, \frac{1}{f}\right)$, $N_k\left(r, \frac{1}{f}\right)$. If $\overline{E}(1, f) = \overline{E}(1, g)$, we denote by $N_{11}\left(r, \frac{1}{f-1}\right)$ the counting function for common simple 1-points of both $f(z)$ and $g(z)$ where multiplicity is not counted.

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In 1998, Wang and Fang [2] (cf. [3]) proved the following therem.

Theorem A Let $f(z)$ be a transcendental meromorphic function, and n, k be tow positive integers with $n \geq k+1$. Then $(f^n)^{(k)} - 1$ has infinitely many zeros.

It is interesting to establish the unicity theorem corresponding to the above result. In 2002, Fang [4] obtained the following result.

Theorem B Let f, g be tow nonconstant entire function, and n, k be tow positive integers with $n(\geq 2k+4)$. If $(f^n)^{(k)}$ and $(g^n)^{(k)}$ share 1 CM, then either $f_{(z)} = c_1 e^{cz}$, $g_{(z)} = c_2 e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(-1)^k (c_1 c_2)^n c^{2k} = 1$, or $f = tg$ for a constant t such that $t^n = 1$.

Recently, Bhoosnurmath and Dyavanal [5] extended Theorem B to the meromorphic case, as follows.

Theorem C Let f, g be tow nonconstant meromorphic function, and n, k be tow positive integers with $n(\geq 3k+8)$. If $(f^n)^{(k)}$ and $(g^n)^{(k)}$ share 1 CM, then either $f_{(z)}=c_1e^{cz}, g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(-1)^k(c_1c_2)^n c^{2k}=1$, or $f=tg$ for a constant t such that $t^n=1$.

Let $k=1$, $f=(n+1)^{-\frac{1}{n+1}}F$ and $g=(n+1)^{-\frac{1}{n+1}}G$ in Theorem C. Then $\left[f^{n+1}\right]'=F^nF'$ and $G\left[g^{n+1}\right]'=G^nG'$. We see that the following result, which is proved by Yang and Hua [6], is a direct consequence of Theorem C.

Theorem D Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, and $n\geq 11$ an integer. If f^nf' and g^ng' share 1 CM, then either $f_{(z)}=c_1e^{cz}$, $g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1c_2)^{n+1}c^2=-1$ or $f=tg$ for a constant t such that $t^{n+1}=1$.

In this paper, we will extend the above result as follows.

Theorem 1 Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, $k(\geq 3)$, $n(\geq 11)$ be tow positive integers. If $E_k(1, f^nf')=E_k(1, g^ng')$, then either $f_{(z)}=c_1e^{cz}, g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1c_2)^{n+1}c^2=-1$ or $f=tg$ for a constant t such that $t_{n+1}=1$.

Theorem 2 Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, $n(\geq 13)$ be a positive integer. If $E_2(1, f^nf')=E_2(1, g^ng')$, then the conclusion of Theorem 1 holds.

Theorem 3 Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, $n(\geq 19)$ be a positive integer. If $E_1(1, f^nf')=E_1(1, g^ng')$, then the conclusion of Theorem 1 holds.

2. Some Lemmas

For the proof of our results, we need the following lemmas.

Lemma 1 [7]. Let f be a nonconstant meromorphic function,and let $a_0, a_1, \cdots, a_n$ be finite complex numbers such that $a_n\neq 0$. Then

$$\begin{aligned}&T\left(r, a_nf^n+a_{n-1}f^{n-1}+\cdots+a_1f^1+a_0\right)\\&=nT(r,f)+S(r,f)\end{aligned}$$

Lemma 2 [6]. Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, $n(\geq 6)$ be a positive integer, if $f^nf'g^ng'=1$ then either $f_{(z)}=c_1e^{cz}$, $g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1c_2)^{n+1}c^2=-1$.

Lemma 3 [8]. Let f be a nonconstant meromorphic function, k a positive integer, then

$$N\left(r,\frac{1}{f^{(k)}}\right)\leq N\left(r,\frac{1}{f}\right)+k\overline{N}(r,f)+S(r,f).$$

Lemma 4 [9]. Let f and g be two nonconstant meromorphic functions,and let k be a positive integer. If $E_k(1,f)=E_k(1,g)$, then one of the following cases must occur:

$$\begin{aligned}T(r,f)+T(r,g)&\leq \overline{N}_2(r,f)+\overline{N}_2\left(r,\frac{1}{f}\right)+\overline{N}_2(r,g)+\overline{N}_2\left(r,\frac{1}{g}\right)+\overline{N}_2\left(r,\frac{1}{f-1}\right)+\overline{N}_2\left(r,\frac{1}{g-1}\right)\\&-N_{11}\left(r,\frac{1}{f-1}\right)+N_{(k+1}(r,\frac{1}{f-1})+N_{(k+1}\left(r,\frac{1}{f-1}\right)+S(r,f)+S(r,g)\end{aligned}\qquad(1)$$

$$f=\frac{(b+1)g+(a-b-1)}{bg+(a-b)}, \text{ where } a(\neq 0), b \text{ are tow constants.}\qquad(2)$$

Lemma 5. Let f and g be two nonconstant meromorphic functions, $n(\geq 6)$ be a positive integer, set $F=f^nf'$, $G=g^ng'$, if

$$F=\frac{(b+1)G+(a-b-1)}{bG+(a-b)}\qquad(2.1)$$

where $a(\neq 0)b$ are two constants, then either $f_{(z)}=c_1e^{cz}$, $g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1c_2)^{n+1}c^2=-1$ or $f=tg$ for a constant t such that $t^{n+1}=1$.

Proof. By Lemma 1, we get

$$\begin{aligned}T(r,F)&=T\left(r,f^nf^{'}\right)\leq T\left(r,f^n\right)+T\left(r,f^{'}\right)\leq nT(r,f)+2T(r,f)+S(r,f)\\&=(n+2)T(r,f)+S(r,f)\end{aligned}\qquad(2.2)$$

$$\begin{aligned} nT(r,f) &= T(r,f^n)+S(r,f) = N(r,f^n)+m(r,f^n)+S(r,f) \\ &\le N(r,f^n f')-N(r,f')+m(r,f^n f')+m\left(r,\frac{1}{f'}\right)+S(r,f) \\ &\le T(r,f^n f')+T(r,f')-N(r,f')-N\left(r,\frac{1}{f'}\right)+S(r,f) \\ &\le T(r,F)+T(r,f)-N(r,f)-N\left(r,\frac{1}{f'}\right)+S(r,f) \end{aligned} \tag{2.3}$$

So,

$$T(r,F) \ge (n-1)T(r,f)+N(r,f)+N\left(r,\frac{1}{f'}\right)+S(r,f) \tag{2.4}$$

Thus, by (2.2) and (2.3), we get $S(r,F)=S(r,f)$. Similarly, we get

$$\begin{aligned} T(r,G) &\ge (n-1)T(r,g)+N(r,g) \\ &+N\left(r,\frac{1}{g'}\right)+S(r,g) \end{aligned} \tag{2.5}$$

$$S(r,G)=S(r,g)$$

It is clear that the inequality $T(r,f)\le T(r,g)$ or $T(r,g)\le T(r,f)$ holds for a set of infinite measure of r.

Without loss of generality, we may suppose that $T(r,f)\le T(r,g)$, holds for $r\in I$, where I is a set with infinite measure. Next we consider five cases.

Case 1. $a\ne b, b\ne 0,-1$,

If $a-b-1\ne 0$, then by the 2.1 we known:

$$\overline{N}\left(r,\frac{1}{F}\right)=\overline{N}\left(r,\frac{1}{G+\frac{(a-b-1)}{b+1}}\right)$$

By the Nevalinna second fundamental theorem and lemma 3, we have

$$\begin{aligned} T(r,G) &\le \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G+\frac{(a-b-1)}{b+1}}\right)+S(r,G) \\ &= \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{F}\right)+S(r,g) \\ &\le \overline{N}(r,g)+\overline{N}\left(r,\frac{1}{g}\right)+N\left(r,\frac{1}{g'}\right)+\overline{N}\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f'}\right)+S(r,g) \\ &\le \overline{N}(r,g)+\overline{N}\left(r,\frac{1}{g}\right)+N\left(r,\frac{1}{g'}\right)+N(r,f)+2N\left(r,\frac{1}{f}\right)+S(r,g) \\ &\le T(r,g)+\overline{N}(r,g)+N\left(r,\frac{1}{g'}\right)+3T(r,f)+S(r,g) \\ &\le 4T(r,g)+\overline{N}(r,g)+N\left(r,\frac{1}{g'}\right)+S(r,g) \end{aligned}$$

By $n\ge 6$ and (2.5), we get $T(r,g)\le S(r,g)$, for $r\in I$, a contradiction.

If $a-b-1=0$, by (2.1) we can obtain:

$$F=\frac{(b+1)G}{bG+1}$$

We see that:

$$\overline{N}(r,F)=\overline{N}\left(r,\frac{1}{G+\frac{1}{b}}\right)$$

Combining the Nevalinna second fundamental theorem and lemma 3, we have

$$T(r,G) \le \overline{N}(r,G) + \overline{N}\left(r,\frac{1}{G}\right) + \overline{N}\left(r,\frac{1}{G+\frac{1}{b}}\right) + S(r,G)$$
$$= \overline{N}(r,G) + \overline{N}\left(r,\frac{1}{G}\right) + \overline{N}(r,F) + S(r,g)$$
$$\le \overline{N}(r,g) + \overline{N}\left(r,\frac{1}{g}\right) + N\left(r,\frac{1}{g'}\right) + \overline{N}\left(r,\frac{1}{f}\right) + S(r,g)$$
$$\le T(r,g) + \overline{N}(r,g) + N\left(r,\frac{1}{g'}\right) + T(r,f) + S(r,g)$$
$$\le 2T(r,g) + \overline{N}(r,g) + N\left(r,\frac{1}{g'}\right) + S(r,g)$$

By $n \ge 6$ and (2.5), we get $T(r,g) \le S(r,g)$, $r \in I$, a contradiction.

Case 2. $a \ne b, b = -1$, So $F = \frac{a}{(a+1)-G}$

We can get $\overline{N}(r,F) = \overline{N}\left(r,\frac{1}{G-(a+1)}\right)$, similarly as Case 1, it is impossible.

Case 3. $a \ne b, b = 0$, So $F = \frac{G+(a-1)}{a}$.

If $a-1=0$, then $F \equiv G$, so $f^n f' \equiv g^n g'$.

It follows that:

$f^n f' \equiv g^n g' F_1 = \frac{f^{n+1}}{n+1} = \frac{g^{n+1}}{n+1} + C = G_1 + C$, where C is a constant.

We state that C is zero. If not, one we can get that from the Nevalinna second fundamental theorem and lemma 1.

$$(n+1)T(r,g) = T(r,G_1) + S(r,g)$$
$$\le \overline{N}(r,G_1) + \overline{N}\left(r,\frac{1}{G_1}\right) + \overline{N}\left(r,\frac{1}{G_1+1}\right) + S(r,g)$$
$$= \overline{N}(r,G_1) + \overline{N}\left(r,\frac{1}{G_1}\right) + \overline{N}\left(r,\frac{1}{F_1}\right) + S(r,g)$$
$$= \overline{N}(r,g) + \overline{N}\left(r,\frac{1}{g}\right) + \overline{N}\left(r,\frac{1}{f}\right) + S(r,g)$$
$$\le 3T(r,g) + S(r,g)$$

Because $n \ge 6$, we can get $T(r,g) \le S(r,g)$, for $r \in I$, which is impossible. So C is zore.

Then $F_1 \equiv G_1$, it gives that $f^{n+1} = g^{n+1}$, so $f = tg$, where t is constant satisfying $t^{n+1} = 1$.

Case 4. $a = b$, $b \ne 0, -1$, from (2.1) we can get $F = \frac{(b+1)G-1}{bG}$ $\overline{N}(r,F) = \overline{N}(r,\frac{1}{G})$, similarly as Case 1, it is impossible.

Since $a \ne 0$, now we consider the following case.

Case 6. $a = b = -1$

It yields $FG \equiv 1$, that is: $f^n f' g^n g' = 1$. By the Lemma 2, we can get $f_{(z)} = c_1 e^{cz}, g_{(z)} = c_2 e^{-cz}$ where c_1, c_2, c are three constants, satisfying $(c_1 c_2)^{n+1} c^2 = -1$.

Now the proof of Lemma 5 is completed.

3. Proof of Theorems

Proof of theorem 1:

Noticing that $k \ge 3$, we have

$$\overline{N}\left(r,\frac{1}{F-1}\right) + \overline{N}\left(r,\frac{1}{G-1}\right) - N_{11}\left(r,\frac{1}{F-1}\right)$$
$$+\overline{N}_{(k+1}\left(r,\frac{1}{F-1}\right) + \overline{N}_{(k+1}\left(r,\frac{1}{G-1}\right)$$
$$\le \frac{1}{2}N\left(r,\frac{1}{F-1}\right) + \frac{1}{2}N\left(r,\frac{1}{G-1}\right)$$
$$\le \frac{1}{2}T(r,F) + \frac{1}{2}T(r,G) + O(1)$$

By lemma 4, we can get

$$\begin{aligned} T(r,F)+T(r,G) &\le 2\left\{N_2\left(r,\frac{1}{F}\right) + N_2(r,F) + N_2\left(r,\frac{1}{G}\right) + N_2(r,G)\right\} + S(r,F) + S(r,G) \\ &= 2\left\{N_2\left(r,\frac{1}{F}\right) + N_2(r,F) + N_2\left(r,\frac{1}{G}\right) + N_2(r,G)\right\} + S(r,f) + S(r,g) \end{aligned} \tag{3.1}$$

Because:

$$N_2\left(r,\frac{1}{F}\right)+N_2\left(r,F\right)=N_2\left(r,\frac{1}{f^n f'}\right)+N_2\left(r,f^n f'\right)\le 2\overline{N}\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f'}\right)+2\overline{N}\left(r,f\right) \quad (3.2)$$

and

$$N_2\left(r,\frac{1}{G}\right)+N_2\left(r,G\right)\le 2\overline{N}\left(r,\frac{1}{g}\right)+N\left(r,\frac{1}{g'}\right)+2\overline{N}\left(r,g\right) \quad (3.3)$$

By (3.1)-(3.3) and lemma 3, we can get:

$$\begin{aligned}T\left(r,F\right)+T\left(r,G\right)&\le 2\left[2\overline{N}\left(r,\frac{1}{f}\right)+2\overline{N}\left(r,f\right)+N\left(r,\frac{1}{f'}\right)+2\overline{N}\left(r,\frac{1}{g}\right)+2\overline{N}\left(r,g\right)+N\left(r,\frac{1}{g'}\right)\right]+S\left(r,f\right)+S\left(r,g\right)\\&=4\overline{N}\left(r,\frac{1}{f}\right)+4\overline{N}\left(r,f\right)+2N\left(r,\frac{1}{f'}\right)+S\left(r,f\right)+4\overline{N}\left(r,\frac{1}{g}\right)+4\overline{N}\left(r,g\right)+2N\left(r,\frac{1}{g'}\right)+S\left(r,g\right)\\&\le 5N\left(r,\frac{1}{f}\right)+5\overline{N}\left(r,f\right)+N\left(r,\frac{1}{f'}\right)+S\left(r,f\right)+5N\left(r,\frac{1}{g}\right)+5\overline{N}\left(r,g\right)+N\left(r,\frac{1}{g'}\right)+S\left(r,g\right)\\&\le 9T\left(r,f\right)+\overline{N}\left(r,f\right)+N\left(r,\frac{1}{f'}\right)+S\left(r,f\right)+9T\left(r,g\right)+\overline{N}\left(r,g\right)+N\left(r,\frac{1}{g'}\right)+S\left(r,g\right)\end{aligned} \quad (3.4)$$

By $n\ge 9$ and (2.4), (2.5) we obtain $T\left(r,f\right)+T\left(r,g\right)\le S\left(r,f\right)+S\left(r,g\right)$, which is impossible.

Therefore, by lemma 4 $f=\dfrac{(b+1)g+(a-b-1)}{bg+(a-b)}$, where $a(\ne 0)$, b are tow constants, it follows by lemma 5 then either $f_{(z)}=c_1e^{cz}, g_{(z)}=c_2e^{-cz}$ where c_1, c_2, c are three constants, satisfying $\left(c_1c_2\right)^{n+1}c^2=-1$ or $f=tg$ for a constant t such that $t^{n+1}=1$.

The proof of Theorem 1 is complete.

Proof of theorem 2:

We can see clearly:

$$\begin{aligned}&\overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)-N_{11}\left(r,\frac{1}{F-1}\right)+\frac{1}{2}\overline{N}_{(3}\left(r,\frac{1}{F-1}\right)+\frac{1}{2}\overline{N}_{(3}\left(r,\frac{1}{G-1}\right)\\&\le\frac{1}{2}N\left(r,\frac{1}{F-1}\right)+\frac{1}{2}N\left(r,\frac{1}{G-1}\right)\le\frac{1}{2}T\left(r,F\right)+\frac{1}{2}T\left(r,G\right)+S\left(r,f\right)+S\left(r,g\right)\end{aligned}$$

By lemma 4, we can get:

$$\begin{aligned}T\left(r,F\right)+T\left(r,G\right)&\le 2\left[N_2\left(r,\frac{1}{F}\right)+N_2\left(r,F\right)+N_2\left(r,\frac{1}{G}\right)+N_2\left(r,G\right)\right]\\&+\overline{N}_{(3}\left(r,\frac{1}{F-1}\right)\overline{N}_{(3}\left(r,\frac{1}{G-1}\right)+S\left(r,f\right)+S\left(r,g\right)\end{aligned} \quad (3.5)$$

Considering

$$\begin{aligned}\overline{N}_{(3}\left(r,\frac{1}{F-1}\right)&\le\frac{1}{2}N\left(r,\frac{F}{F'}\right)=\frac{1}{2}N\left(r,\frac{F'}{F}\right)+S\left(r,f\right)\le\frac{1}{2}\overline{N}\left(r,\frac{1}{F}\right)+\frac{1}{2}\overline{N}\left(r,F\right)+S\left(r,f\right)\\&\le\frac{1}{2}\left[\overline{N}\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f'}\right)+\overline{N}\left(r,f\right)\right]+S\left(r,f\right)\le 2T\left(r,f\right)+S\left(r,f\right)\end{aligned} \quad (3.6)$$

Similarly, we can get

$$\overline{N}_{(3}\left(r,\frac{1}{G-1}\right)\le 2T\left(r,g\right)+S\left(r,g\right) \quad (3.7)$$

By from (3.4)-(3.7), we can get

$$T\left(r,F\right)+T\left(r,G\right)\le 11T\left(r,f\right)+\overline{N}\left(r,f\right)+N\left(r,\frac{1}{f'}\right)+S\left(r,f\right)+11T\left(r,g\right)+\overline{N}\left(r,g\right)+N\left(r,\frac{1}{g'}\right)+S\left(r,g\right)$$

Since $n \geq 13$ and (2.4), (2.5), we can get $T(r,f)+T(r,g) \leq S(r,f)+S(r,g)$ impossible The proof of Theorem 2 is complete.

Proof of theorem 3:

Since:

$$\overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)-N_{11}\left(r,\frac{1}{F-1}\right) \leq \frac{1}{2}N\left(r,\frac{1}{F-1}\right) + \frac{1}{2}N\left(r,\frac{1}{G-1}\right) \leq \frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,f)+S(r,g)$$

We can see clearly from lemma 4 that:

$$\begin{aligned} T(r,F)+T(r,G) &\leq 2\left\{N_2\left(r,\frac{1}{F}\right)+N_2(r,F)+N_2\left(r,\frac{1}{G}\right)+N_2(r,G)+\overline{N}_{(2}\left(r,\frac{1}{F-1}\right)+\overline{N}_{(2}\left(r,\frac{1}{G-1}\right)\right\} \\ &\quad +S(r,F)+S(r,G) \\ &= 2\left\{N_2\left(r,\frac{1}{F}\right)+N_2(r,F)+N_2\left(r,\frac{1}{G}\right)+N_2(r,G)+\overline{N}_{(2}\left(r,\frac{1}{F-1}\right)+\overline{N}_{(2}\left(r,\frac{1}{G-1}\right)\right\} \\ &\quad +S(r,f)+S(r,g) \end{aligned} \tag{3.8}$$

Considering

$$\begin{aligned} \overline{N}_{(2}\left(r,\frac{1}{F-1}\right) &\leq N\left(r,\frac{F}{F'}\right)=N\left(r,\frac{F'}{F}\right)+S(r,f) \leq \overline{N}(r,F)+\overline{N}\left(r,\frac{1}{F}\right)+S(r,f) \\ &\leq N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f'}\right)+\overline{N}(r,f)+S(r,f) \leq 4T(r,f)+S(r,f) \end{aligned} \tag{3.9}$$

Similarly, we can get

$$\overline{N}_{(2}\left(r,\frac{1}{G-1}\right) \leq 4T(r,g)+S(r,g) \tag{3.10}$$

By from (3.8)-(3.10), we can get

$$T(r,F)+T(r,G) \leq 17T(r.f)+\overline{N}(r,f)+N\left(r,\frac{1}{f'}\right)+S(r,f)+17T(r.g)+\overline{N}(r,g)+N\left(r,\frac{1}{g'}\right)+S(r,g)$$

Since $n \geq 19$ and (2.4), (2.5), we can get $T(r,f)+T(r,g) \leq S(r,f)+S(r,g)$, impossible The proof of Theorem 3 is complete.

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