British Journal of Applied Science & Technology
6(4): 350-363, 2015, Article no.BJAST.2015.094
ISSN: 2231-0843

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Discussion on Using Only One Linear Array to Estimate the Phase Velocity of Rayleigh Wave Based on Microtremor Survey

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Authors' contributions

This work was carried out in collaboration between all authors. Authors XZ and HM thought of the initial idea. Author XZ carried out background study and built the methodology. Author XZ carried out the numerical simulation. Authors HM and XZ conducted field tests. Authors XZ carried out the data process and the analysis. Author XZ drafted the manuscript and author HM did the modifications. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJAST/2015/14961
Editor(s):
(1) Vyacheslav O. Vakhnenko, Division of Geodynamics of Explosion, Subbotin Institute of Geophysics, National Academy of Sciences of Ukrainian, Ukraine.
Reviewers:
(1) Jorge Luis de Souza, Coordenação de Geofísica, MCTI/Observatório Nacional, Brazil.
(2) Anonymous, China.
(3) Anonymous, USA.
(4) Anonymous, Egypt.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=767&id=5&aid=7475

Original Research Article

Received 29th October 2014
Accepted 3rd December 2014
Published 26th December 2014

ABSTRACT

Aims: To propose a new method of estimating the phase velocity of Rayleigh wave using one linear array based on the spatial auto-correlation (SPAC) method and complex coherence function (CCF) and confirm its availability and robustness using both the numerical simulation and a field test.
Study Design: Cross-sectional study.
Place and Duration of Study: Department of Built Environment, Tokyo Institute of Technology and Zoorasia Yokohama Zoological Gardens (ZRS), between August 2013 and July 2014.
Methodology: Numerical simulation was conducted to test the behavior of the proposed method using linear arrays with different directions in different kind of azimuth-dependent microtremor wave field; Field test was carried out at the parking lot of ZRS, in which the proposed method was applied with 2 linear arrays with different direction and the estimation of phase velocity was

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compared with that from the SPAC method and theoretical one. Through the numerical simulation and field test, the SPAC method without taking azimuthal average (we call it J_0 method in the text) was also conducted for comparison.
Results: In the numerical simulation, the estimations of phase velocity from linear arrays with different directions are accurate and coincide with each other except for some extreme case; In the field test, the estimations of the 2 linear arrays both match well with the theoretical one and that from SPAC method. On the other hand, the SPAC method without azimuthal average behaved badly and showed instability because of its intrinsic defect.
Conclusion: By applying the proposed method, it is available to estimate the phase velocity of Rayleigh wave using just one linear array in a microtremor wave field, which is not a strongly azimuth-dependent one.

Keywords: SPAC method; CCF; linear array; phase velocity.

1. INTRODUCTION

In order to estimate earthquake ground motion, it is necessary to model the ground structure by geophysical exploration. In the field of geophysical exploration, the extensive use of array method has been proposed by employing microtremors because they provide an inexpensive means of inferring the phase velocity of surface waves. Among those methods, the frequency-wave number (f-k) spectral method [1,2] and the semblance analysis [3,4] both require a rather large number of sensors to be settwo-dimensionally although their arrangement can be designed freely to some extent.

In the spatial auto-correlation (SPAC) method [5-7], all the whole information on the wave field is integrated into a single quantity, which is called the azimuthal average of the spatial autocorrelation function. Its theory is simple and it requires fewer sensors than the f-k method or a semblance analysis. Normally, we analyze records from a circular array of evenly spaced sensors on a circle and a central sensor. However, the strict arrangement of the sensors is difficult to realize especially in urban areas. Research has been carried out in order to reduce the restriction of the sensor arrangement [8-11].

The validity of performing the SPAC method with a linear array has been discussed but the conclusion is empirical and is not clearly backed by physical and mathematical theory [9]. Another method, called the direct estimation method (DEM), has been proposed wherein the sensor arrangement is more flexible [10]. In DEM, a complex coherence function (CCF) has been used to constitute a new solution for estimating the phase velocity. However, DEM requires at least five sensors and they must be in a symmetrical arrangement. The possibility of performing ground survey using a linear array is also realized by using refraction microtremor (ReMi) method [12]. This method is proud for not using active sources and its robustness against the presence of noise and full-wave. However, ReMi method uses refraction recording equipment, whose lower frequency limit is confined, and it uses much more sensors (over 12) than normal microtremor survey method to obtain redundant statistics. The phase velocity of Rayleigh wave is decided by picking the lowest velocity envelope. Hence, although this method uses simple form of array, we still consider it improvable because of the large number of sensors and the deficiency in mathematical basis.

On the other hand, we herein propose a technique based on the CCF for estimating the Rayleigh wave phase velocity. With the help of a genetic algorithm (GA) [13], the phase velocity of Rayleigh wave can be estimated from the CCFs among the sensors in a line (linear array). This makes the arrangement of the sensors easier. Besides, because this method is based on the concept of SPAC method, it has the potential to estimate deeper structure using just fewer sensors and with firmer theoretical basis than ReMi method.

This technique is unavailable when the microtremor is uni-directional (all the wave power comes from only one direction from remote place) and perpendicular to the linear array. This is obvious because in this case the microtremor field cannot produce phase difference between any pair of sites in the linear array. However, in real cases, there is barely extreme case like that. In most cases, only one linear array is enough to estimate Rayleigh wave phase velocity regardless of the azimuth-dependence of the microtremor field. On the other hand, this technique, of course, works in the isotropic wave field. For SPAC method without taking azimuthal average, using only 2 sites can also obtain acceptable accuracy in the isotropic wave field [8,9,14] but it does not work in the case where the wave field has some azimuth-dependent component. However, by using 3 sites on a line in the proposed method, phase velocity can be estimated well even in (moderately) azimuth-dependent wave field. Hence, though the proposed method is not as perfect as the original SPAC method, it is expected to have more simple and convenient sensor arrangement and have robustness against azimuth-dependent wave field to certain extent.

We did necessary numerical simulations to demonstrate that (1) this technique works as long as the wave field is not a strong azimuth-dependent one; (2) it works even in some cases of extremely azimuth-dependent wave field. The availability of the SPAC method without taking average was also discussed as comparison. Furthermore, we confirmed the availability of this technique using data from a field test.

2. THEORETICAL BACKGROUND

In this section, we firstly interpret the concept of the CCF first. Then, we demonstrate, in theory, how to estimate the phase velocity of Rayleigh waves from the records of linear arrays by applying CCFs among sensors on a line.

2.1 Discrete Formula of CCF

Under assumptions: (1) Only the fundamental mode of Rayleigh waves is dominated in microtremors, and (2) Different sources are not correlated, the real part of the discrete formula for CCF can be expressed as [10]:

$$Re(\gamma_{pq}) = J_0(kr) + 2\sum_{n=1}^{\infty}\{(-1)^n J_{2n}(kr)\sum_{l=1}^{L}\lambda_l \cos 2n\theta_l\} \quad (2.1)$$

In which $Re$and J_n denote the real part and the *n*-th order Bessel function with the 1st mode. *pq* is the CCF between sites *p* and *q*. *r* is the distance between sensors *p* and *q*. *L* is the number of wave sources. λ_l is the rate of the contribution of the l-th wave source to the power spectra at the observation point ($\sum_{l=1}^{L}\lambda_l$ = 1). k is the wave number and θ_l is the azimuth of the wave source *l* (see Fig. 1).

In the conventional SPAC method [6],

$$\rho(\omega) = \frac{1}{3}\sum_{y=1}^{3}\frac{Re\left(S_{0y}(\omega)\right)}{\sqrt{S_{00}(\omega)S_{yy}(\omega)}} \quad (2.2)$$

Is commonly used. $\rho(\omega)$ is the azimuthal average of the spatial autocorrelation coefficient. $S_{yy}(\omega)$ and $S_{0y}(\omega)$ are the power spectra of the vertical component of the microtremors at site *y*(*y*= 1, 2, 3, 4) and the cross spectra of the vertical records between site *y* and 0, respectively. Site 0 is the center of the array, and sites 1, 2, and 3 are located on the circle (see Fig. 2a). Here, $\frac{Re\left(S_{0y}(\omega)\right)}{\sqrt{S_{00}(\omega)S_{yy}(\omega)}}$is actually equal to $Re(\gamma_{0y})$. By taking the azimuthal average, the Bessel functions of orders 2 and 4 vanish and the Bessel functions of orders larger than 6 are negligibly smaller than $J_0(kr)$ in the range of $kr < \pi$ sothat only one term: $J_0(kr)$ remains. Then, the phase velocity v ($v = \frac{\omega}{k}$) (can be obtained using certain inversion techniques.

2.2 A Proposed Method Using a Linear Array on the Basis of the CCF

In the range of $kr < \pi$, the Bessel functions of orders larger than 6 can be ignored [10]. Hence, the real part of the CCF is expressed as:

$$Re(\gamma_{pq}) \approx J_0(kr) - 2J_2(kr)\sum_{l=1}^{L}\lambda_l \cos 2\theta_l + 2J_4(kr)\sum_{l=1}^{L}\lambda_l \cos 4\theta_l \quad (2.3)$$

In the SPAC method, as interpreted in the previous subsection, the second and third terms vanish when taking the azimuthal average so that the information on wave sources $\sum_{l=1}^{L}\lambda_l \cos 2\theta_l$ and $\sum_{l=1}^{L}\lambda_l \cos 4\theta_l$ do not need to be considered.

On the other hand, we let $\sum_{l=1}^{L}\lambda_l \cos 4\theta_l$ and $\sum_{l=1}^{L}\lambda_l \cos 4\theta_l$ remain and regard them as thesecond and third unknown variables to be estimated in addition to the phase velocity. It is impossible to solve them using just one single CCF. Hence, we propose a linear array to solve this problem. Suppose there are 3 sites A, B and C in a line (see Fig. 2b), there are 3 CCFs, namely, γ_{AB}, γ_{BC}, and γ_{AC}, respectively. The advantage of linear array is that the azimuth of the *n-th* source, θ_l, is thought to be the same for each CCF so that the three CCFs share the same three unknown variables.

$$\begin{cases} Re(\gamma_{AB}) \approx J_0\left(r_{AB}\frac{\omega}{v}\right) - 2J_2\left(r_{AB}\frac{\omega}{v}\right)\sum_{l=1}^{L}\lambda_l \cos 2\theta_l + 2J_4(r_{AB}\frac{\omega}{v})\sum_{l=1}^{L}\lambda_l \cos 4\theta_l. \\ Re(\gamma_{AC}) \approx J_0(r_{AC}\frac{\omega}{v}) - 2J_2(r_{AC}\frac{\omega}{v})\sum_{l=1}^{L}\lambda_1 \cos 2\theta_l + 2J_4(r_{AC}\frac{\omega}{v})\sum_{l=1}^{L}\lambda_l \cos 4\theta_l. \\ Re(\gamma_{BC}) \approx J_0(r_{BC}\frac{\omega}{v}) - 2J_2(r_{BC}\frac{\omega}{v})\sum_{l=1}^{L}\lambda_l \cos 2\theta_l + 2J_4(r_{BC}\frac{\omega}{v})\sum_{l=1}^{L}\lambda_l \cos 4\theta_l. \end{cases} \quad (2.4)$$

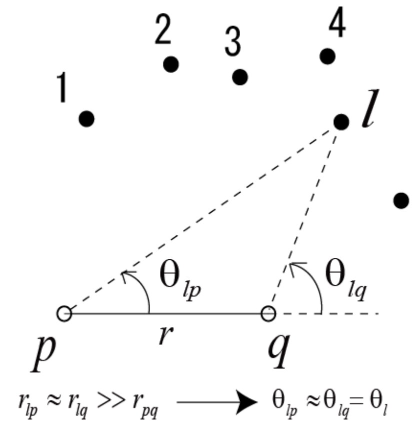

Fig. 1. Geometry used in the formulation of complex coherence function (CCF) between two sites p and q

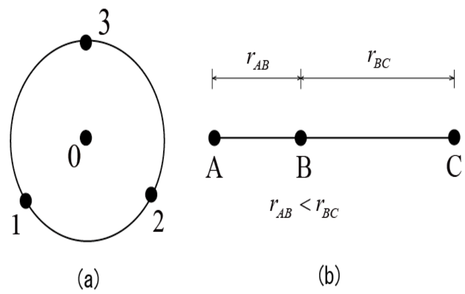

Fig. 2. Sensor arrangement of SPAC method and the new method (a) The equilateral array used by SPAC method (b) The linear array with 3 sites on it

According to Equation (2.4), we obtain γ_{AB}, γ_{AC} and γ_{BC} from the observation records as Knowns. Then we aim to estimate the 3 unknowns, namely, v, $\sum_{l=1}^{L}\lambda_l \cos 2\theta_l$, and $\sum_{l=1}^{L}\lambda_l \cos 4\theta_l$. Under the assumption

of $kr < \pi$, the CCF with the largest *r* determines the effective range for this method.

For example, in the case shown in Fig. 2b, the effective scope is $k < \frac{\pi}{r_{AC}}$.

To find out the optimum solution, we apply a GA

[13] as the inversion technique to fit each CCF and to estimate the three unknowns $v, \sum_{l=1}^{L}\lambda_l \cos 2\theta_l$, and $\sum_{l=1}^{L}\lambda_l \cos 4\theta_l$. For convenience in applying the GA, we use a 30-bit long integer to indicate the 3 unknowns each of which is 10-bit long, namely,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} kr_{max} \\ \sum_{l=1}^{L}\lambda_l \cos 2\theta_l \\ \sum_{l=1}^{L}\lambda_l \cos 4\theta_l \end{bmatrix} \quad (2.5)$$

Here, $x_1$is chosen to be $kr_{\max}$ in which r_{max} is the largest array interval. Hence, we set $0 < x_1 < \pi$so that x_1 has the resolution of $\frac{\pi}{2^{10}}$. $-1 < x_2 < 1$ so that x_2 has the resolution of $\frac{2}{2^{10}}$and the samefor x_3. If we define:

$$g(x, PQ) = Re(\gamma_{PQ}) - J_0\left(\frac{r_{PQ}}{r_{\max}}x_1\right) + 2J_2\left(\frac{r_{PQ}}{r_{\max}}x_1\right)x_2 - 2J_4\left(\frac{r_{PQ}}{r_{\max}}x_1\right)x_3 \quad (2.6)$$

where PQ indicates the 2 sites, namely, AB, AC, and BC, the fitness function is defined as:

$$\text{fitness} = \exp\left(\frac{-[g(x,AC)^2 + g(x,AB)^2 + g(x,BC)^2]}{2 \times 0.05^2}\right)/(\sqrt{2\pi} \times 0.05) \quad (2.7)$$

Good estimation is expected by setting the population to be 50 and the evolution times to be 200.

3. NUMERICAL SIMULATION

In this section, we use a numerical simulation to create a simple wave field composed of plane waves. We demonstrate the process of using a GA to estimate the phase velocity and to examine the accuracy of the estimation in different wave fields.

3.1 Methodology

In order to constitute required wave field, firstly uni-directional plane waves is numerically simulated. We assume there is a linear array with 3 sensors, namely, A, B, and C. The intervals between adjacent sensors (*r* and 0.5*r*, *r* = 30*m*) are set to be different so that we can have 3 totally different CCFs (γ_{AB}, γ_{AC} and γ_{BC}) tosolve out the optimum solution. The vertical record for site A is constituted by:

$Z(t) = \sum_{i=1}^{n}[\, a(i)\cos(i\omega_0 t) + b(i)\sin(i\omega_0 t)]$(3.1)

Where in $a(i)$ and $b(i)$ are given by realizations of independent Gaussian random numbers. The timestep is set to be 512 and the time interval is 0.08s, so the$\omega_0 = \frac{2\pi}{512\times 0.08s}$.

The fundamental mode of the phase velocity of Rayleigh waves is assumed (see Fig. 3). Hence, under the assumption of plane waves and given the direction of the unidirectional wave, records for the other two sites can be calculated.

Using these records, the CCFs between each pair of sites can be obtained. The effective scopeis $k < \frac{\pi}{r_{AC}} = \frac{\pi}{45m}$, namely, $\frac{2f}{v} < \frac{1}{45}$. Given the assumed phase velocity (see Fig. 3), the effective scopeis around $f < 4.5Hz$. Then, using the inversion technique introduced previously, the estimated phase velocity can be obtained for different cases of a synthetic wave field.

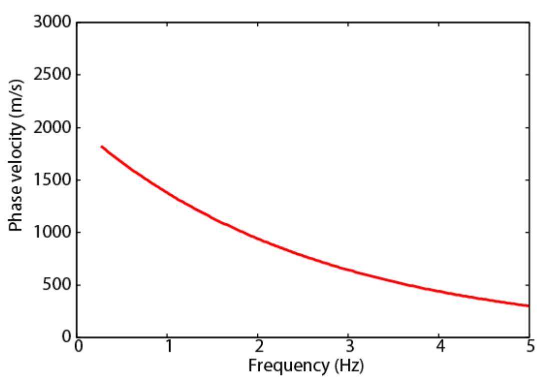

Fig. 3. The fundamental mode of phase velocity of Rayleigh wave assumed for the numerical simulation

3.2 Results and Discussion

3.2.1 For an extremely azimuth-dependent wave field

For an extremely azimuth-dependent wave field, the wave is just uni-directional. We see the estimation of the phase velocity in the case of uni-directional waves in different directions (Fig. 4). The middle part of the figure shows the estimation of the phase velocity (obtained fromx_1). The right partshows the estimations of x_2 and x_3, wherein the solid line shows the theoretical values for x_2 and x_3.In the case of $0°$, $30°$ and $45°$ (Fig. 4a), (b), and (c)), the estimations matches the theoretical ones well in the frequency range of 2.0 to $4.0Hz$, or in the kr_{max} range of 0.5 to 2.0, wherein r_{max}denotesthe largest interval among the sites. The estimation is poor when the angle is larger than $45°$(Fig. 4d), and (e)). To understand this, we see how the CCF varies with x_1 in case of different x_2 and x_3(Fig. 5). It demonstrates that the CCFs are more sensitive with respect to x_1 when x_2 is closer to 1 and/or x_3 is closer to -1. Hence, it is clear that in the case of $0°(x_2 = 0,$ $x_3 = -1)$, $30°(x_2 = 0.5, x_3 = -0.5)$, and $45°(x_2 = 1,\, x_3 = 1)$, the sensitivity with respect to x_1 is higher than $60°$ ($x_2 = -0.5$, $x_3 = -0.5$) and $80°$ ($x_2 = -0.94$, $x_3 = 0.77$). Moreover, since $J_2(kr)$ is larger than $J_4(kr)$ when$kr < \pi$(Fig. 6), the sensitivity of CCFs with respect to $x_2$is higher than that with respect to x_3.This explains that in the case of $0°(x_2 = 0,\, x_3 = -1)$, $30°(x_2 = 0.5,$ $x_3 = -0.5)$ and $45°(x_2 = 1, x_3 = 1)$, the estimation of x_3 is not accurate enough (Fig. 4a), (b), and (c)). From the numericalsimulation above, we can draw a conclusion that in such an extremely azimuth-dependent wave field, the linear array method works when the angle between the dominant wave and the linear array is at least smaller than $60°$.

Ch´avez-Garc´ıa et al. [8,9] have advocated that the SPAC method without taking azimuthal average could provide acceptable accuracy in certain kind of wave fields. This means that only J_0 term of Equation (2.1) is enough to estimate the phase velocity of Rayleigh using a simultaneous observation at only two sites, thus, we call their concept "J_0 method", hereafter. However, it is strictly available only under the condition of isotropic wave field [7,14]. In the extremely azimuth-dependent wave field as described above, the J_0 method is applied using BC and the estimation is also shownin Fig. 4 (the brown dots). We can see that in case (d) and (e), it has quite bad match similar asthat from the proposed method, as expected. However, for case (a) and (b), the estimations deviate from the theoretical one while estimation from the proposed method is quite good. This is because the J_0 method does not consider the effect of the J_2 and J_4 terms. Considering the contributionof J_4term is relatively much smaller than J_2 (Fig. 5), the absolute ratio between J_2 term and J_0 *term*:

$$R_{J_2/J_0} = \frac{|2x_2 J_2(kr)|}{|J_0(kr)|} \qquad (3.2)$$

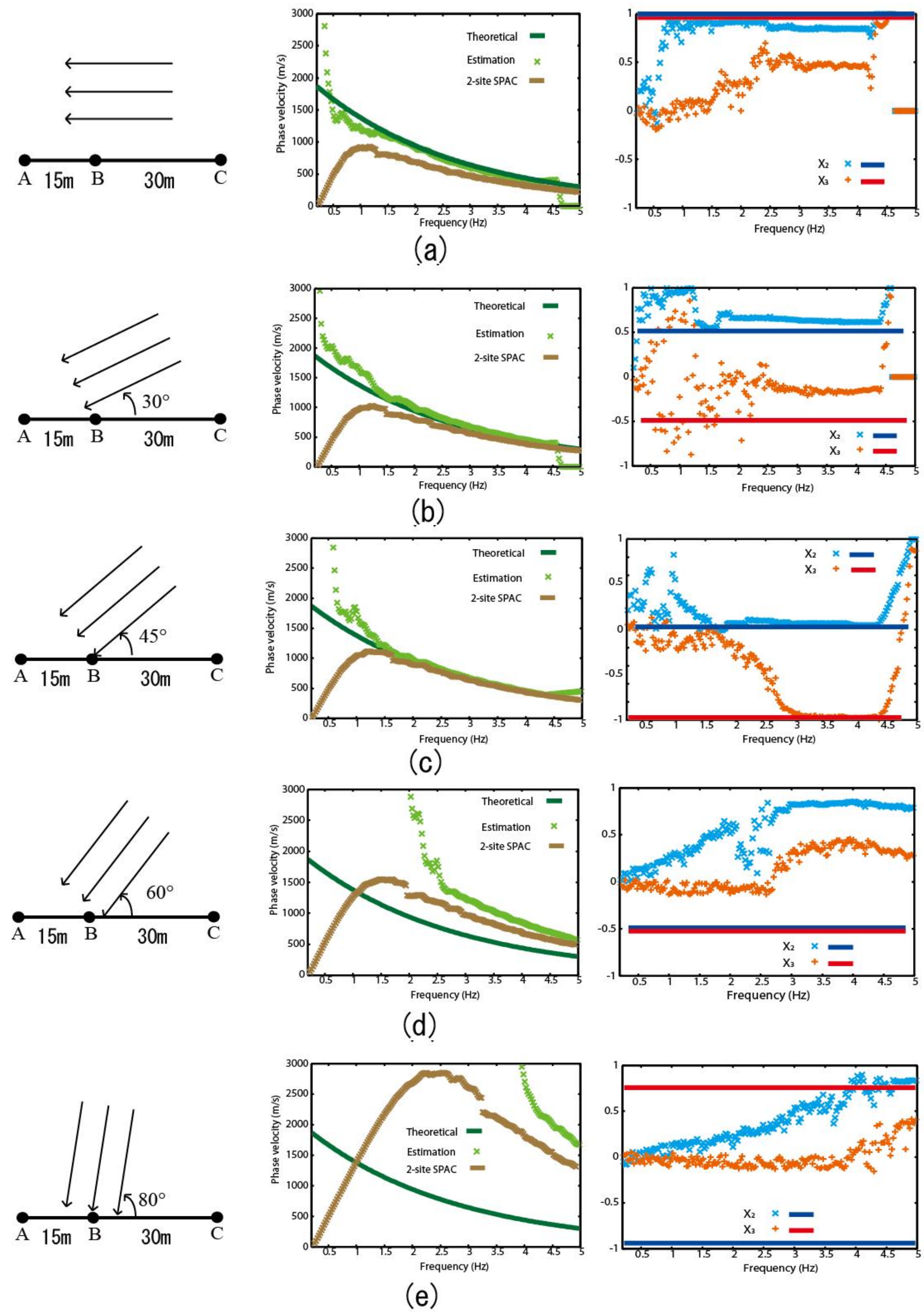

Fig. 4. Estimation of phase velocity, x_2 and x_3 in case of uni-directional plane wave with different direction. (a) 0°(b) 30°(c)45°(d) 60°(e) 80°

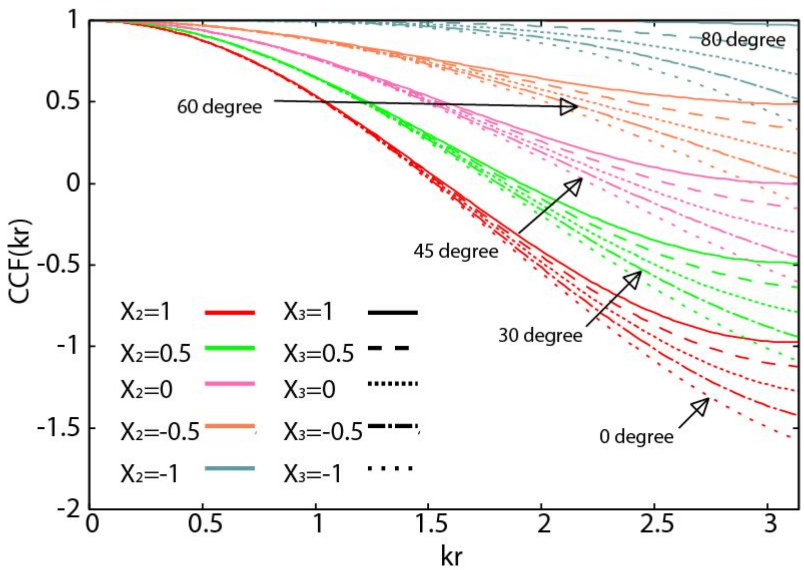

Fig. 5. Variation of CCF with respect to krin case of different x_2 and x_3 (Equation 2.3)

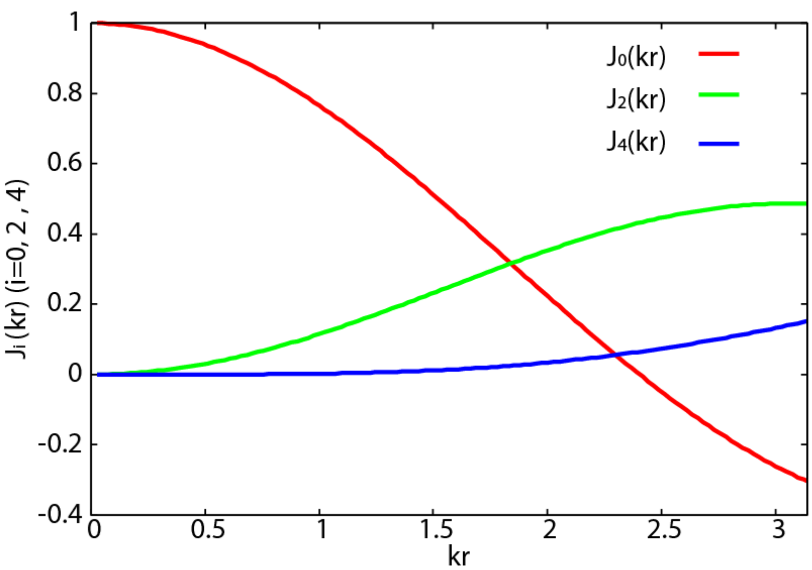

Fig. 6. The variation of bessel function (of order 0, 2, and 4) with respect to
It demonstrates the sensitivity of CCF with respect to x_2 and $x_3$in equation (2.3)

Is shown in Fig. 7. In the figure, we show the variation of the contribution with respect to the angle between linear array and the uni-directional wave. We can see that when the degree is $0°$, namely, in case (a), the contribution of J_2 term is very large. This explains the reason of thedeviation of estimation from J_0 method. Moreover, in case (c) $(45°)$, the contribution of J_2 term is thesmallest so the deviation is almost zero. It shows another possible case that J_0 method can be well applied except for the isotropic wave field.

3.2.2 For a moderately azimuth-dependent wave field

In real cases, there is barely such extremely azimuth-dependent wave field. Past research has shown that when in low-frequency range there is some azimuth-dependent component, in high-frequency range, the wave field is almost isotropic or moderately azimuth-dependent [15]. We demonstrate that the proposed method works well in such wave field. We use the same technique to constitute this kind of wave field, for simplicity, in all-frequency range. As Fig. 8

shows, it is an isotropic wave field where waves come from 36 directions evenly (one wave for every 10 degrees). As Fig. 9 shows, it is a moderately azimuth-dependent wavefield. The only difference with the isotropic one is that the wave from one particular direction has power 10 times larger than wave from other directions. For both wave fields, there are four linear arrays with different directions. The estimation from these linear arrays is shown in the same figures.

We can see that for both wave fields, the estimations are accurate in the frequency range of $2.5$to $4.5Hz$, or in the kr_{max}range of 0.8 to 3.0. It confirms the availability of the proposed method inthe isotropic wavefieldand in the moderately azimuth-dependent wave field. In these wave fields, the estimation from one linear array is stable regardless of azimuth.

4. A FIELD TEST

4.1 Observation

We have applied the proposed technique to field tests in order to confirm the availability of the method. A field test was conducted on 23 October 2013 in the parking lot of Zoorasia Yokohama Zoological Gardens (ZRS) in Yokohama City, Japan (Fig. 10a). There is one KiK-net site just nearby and the soil profile is available as shown in (Fig. 10b). We deployed 7 seismometers (KVS-300, moving-coil-type velocity sensors with a 2-Hz natural period) constituting two linear arrays (Fig. 10c). The two linear arrays form an angle of $60°$so that the SPAC method could be applied for comparison. For both linear arrays, there are 4 sites with the intervals of 5 m, 15 m and 38 m. For the SPAC method, observations with different array radii (5 m, 20 m and 58 m) could be conducted.

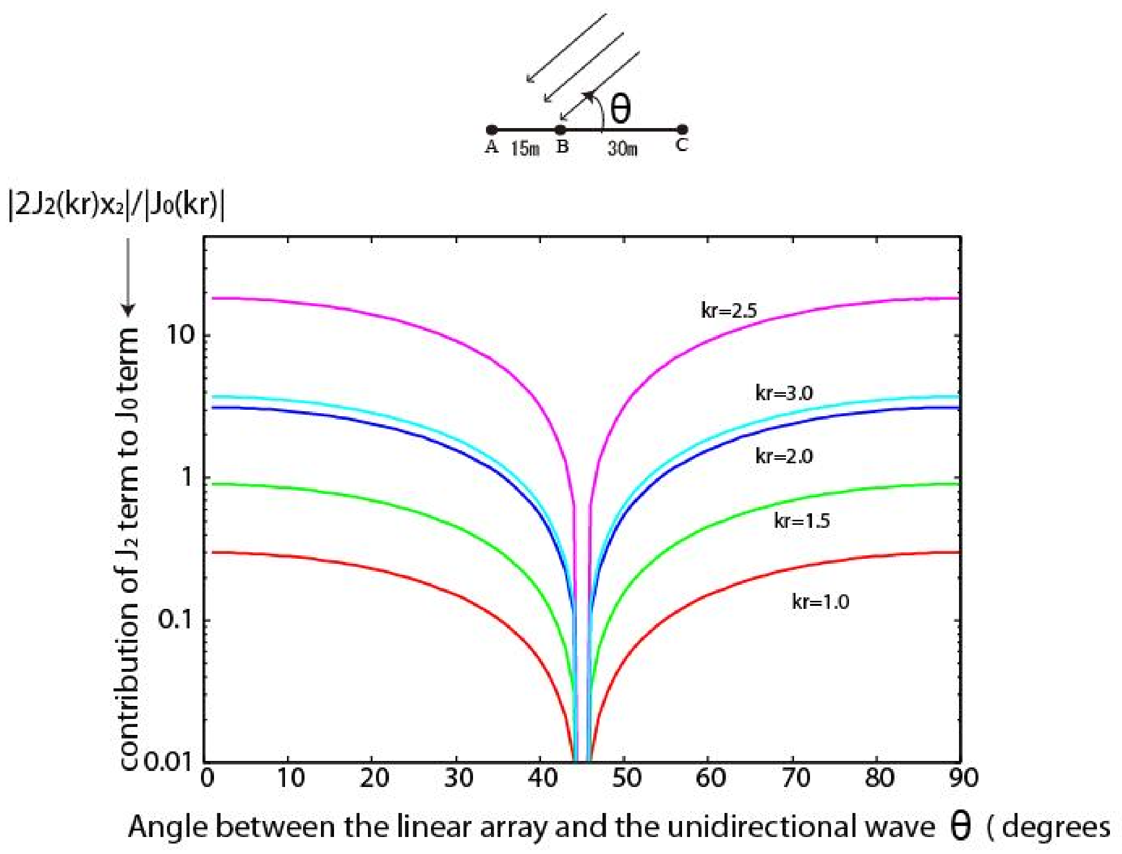

Fig. 7. Contribution of J_2 termtoJ_0 term defined in Equation (3.2)

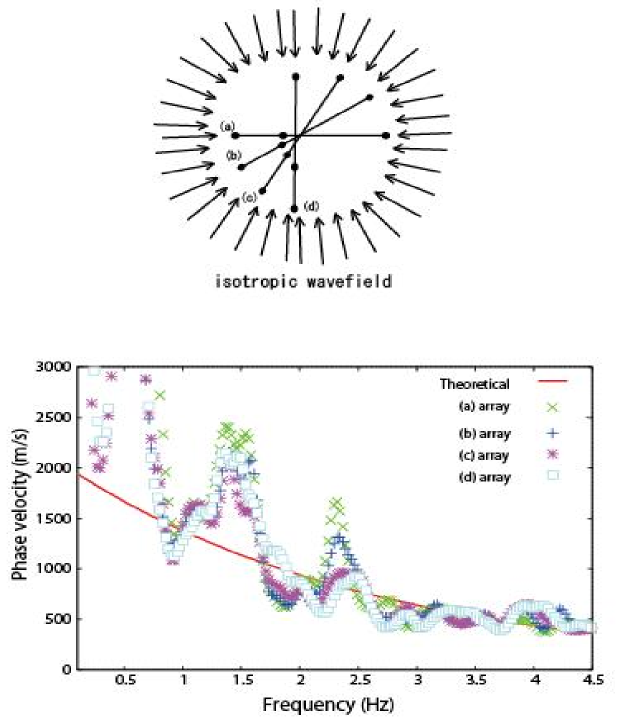

Fig. 8. Estimation from 4 linear arrays with different direction in an isotropic wave field

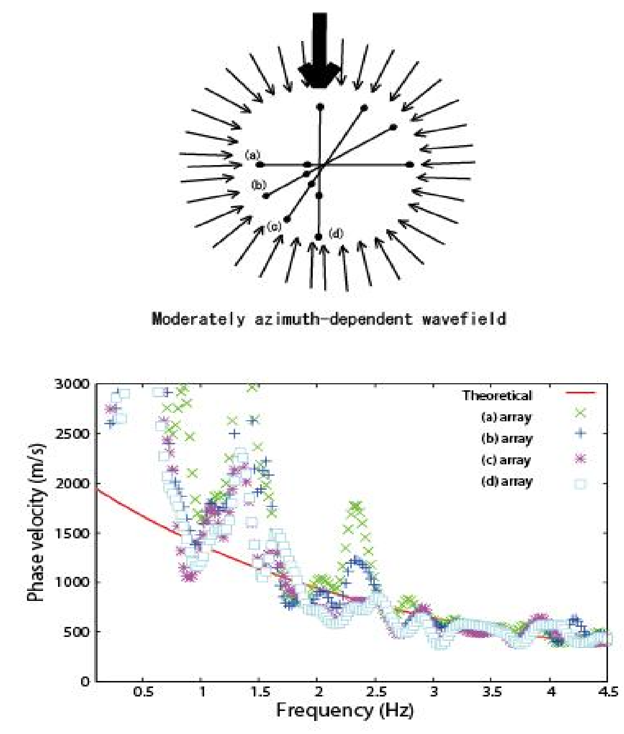

Fig. 9. Estimation from 4 linear arrays with different direction in a moderately azimuth-dependent wave field

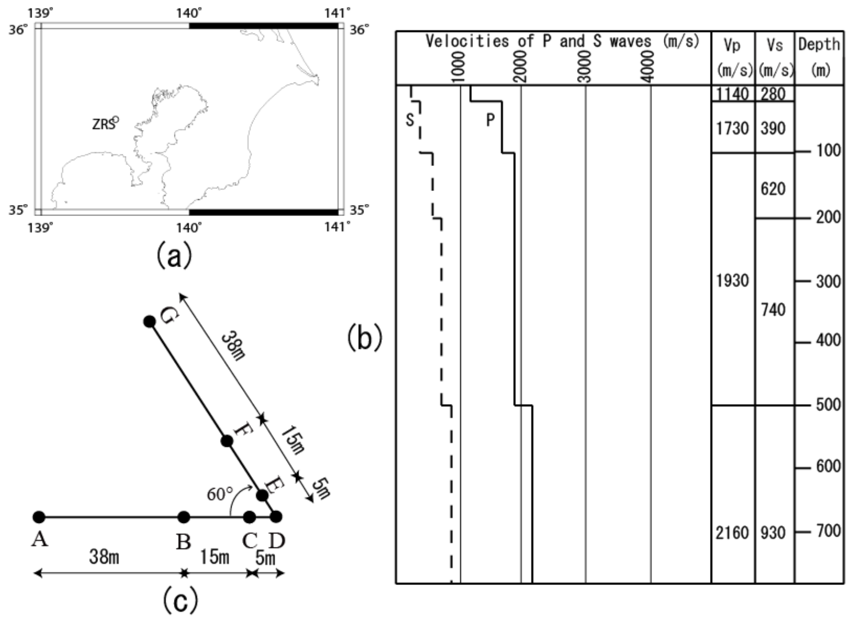

Fig. 10. Field tests. (a) Location of the field tests (b) Boring soil data (c) Arrangement of sensors

The array sizes are decided by considering both the need for SPAC method and proposed method. According to the theoretical dispersion curve shown in Fig. 11 (red line), we set our interested frequency range as 1.0 to 8.0 Hz (700 to 40 m in wave lengh), which is fit for microtremor observation. According to the effective wavelength range of SPAC method (2r to 10r, in which r is the array radius), we set the largest array size as 70 m ($\frac{700}{10}$) and the smallest array size as 20 m ($\frac{40}{2}$). However, the biggest size is limited by the real size of the parking lot so we set the largest size as 58 m. However, if we conduct the proposed method using the intervals of 38 m and 20 m, the effective range would be limited by the largest interval of 58 m, which is expected to be too narrow. Hence, we set another site C (E) between BD (DF). The intervals are set to be 5m and 15m, which have an appropriate difference because we want 3 CCF with different intervals. For the influence of the interval setting on the estimation accuracy, it is not the main point of this article. We consider it as another topic in the future.

For the proposed method, for simplicity, we only choose BD as the linear array with intervals of *BD, BC* and *CD*. Correspondingly, we choose FD as the second linear array. For J_0 method, we chose BD and FD correspondingly. Measurements were conducted over a duration of 30 minutes at a sampling rate of $200Hz$.

4.2 Results and Discussion

We extracted a certain number of segments from the seismograms, worth $40.96s$, which have afiltered range of 0.2 to $10.0Hz$. By processing the records in this field test, we found the noise haslarge influence above $7Hz$ so the estimation of frequency range 0.2 to $7.0Hz$ has good reliability. Then, we took theaverage of the cross-spectra and the power spectra and smoothed them with a Parzen window witha bandwidth of $0.2Hz$. The resulting cross spectra and power spectra are substituted into Equation (2.2) to apply the SPAC method and into Equation (2.6) and Equation (2.7) to apply the proposed method. The results are shown in Fig. 11 with analytical dispersion curve obtained from the soil profile (Fig. 10b). The results from J_0 method using BD and FD, respectively, are shown in Fig. 12. Besides, in order to see the similarities between each estimation and the theoretical phase velocity, the absolute error for each estimation is shown in Fig. 13 and the mean of the error over all frequency range ($0.5\ to\ 8.0Hz$) is calculated in Table 1. Also, we calculated the absolute difference between the estimation from BD and FD in both methods to show the robustness of each method. The result is shown in Fig. 14 and the corresponding mean over all frequency range is calculated in Table 2.

Table 1. The mean over all frequency range of the absolute difference between the estimated phase velocity and the theoretical velocity in each estimation

	BD (linear)	FD (linear)	BD(J_0)	FD(J_0)
Absolute mean (m/s)	177.80	185.64	183.37	211.01

Table 2. The mean over all frequency range of the absolute difference between the estimated phase velocity from BD and FD in proposed method and J_0 method

	Proposed method	J_0 method
Absolute mean (m/s)	43.17	146.41

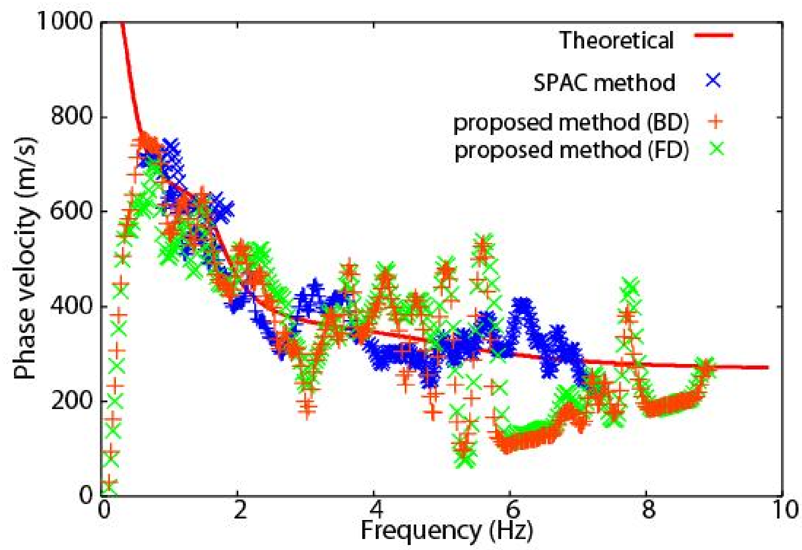

Fig. 11. Comparison between estimation of phase velocity using SPAC method and estimation using the proposed method in the field test

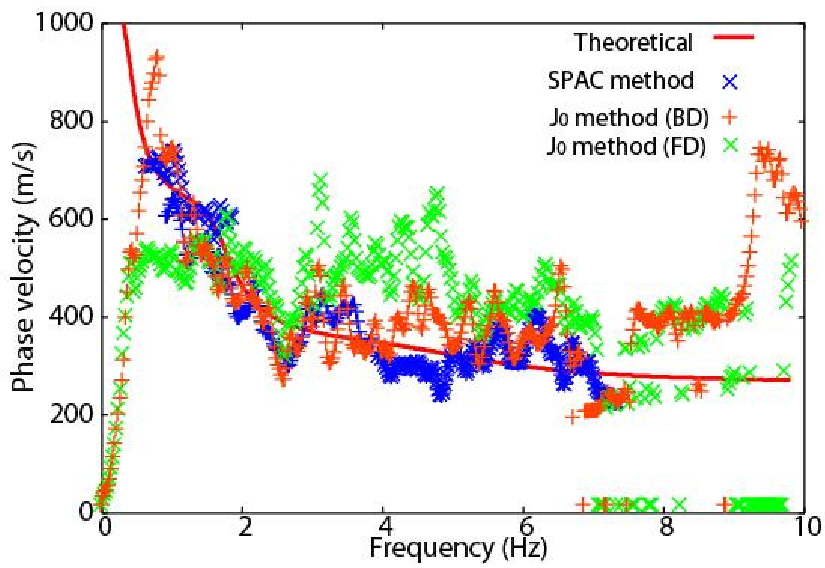

Fig. 12. Comparison between estimation of phase velocity using SPAC method and estimation using the J_0 method in the field test

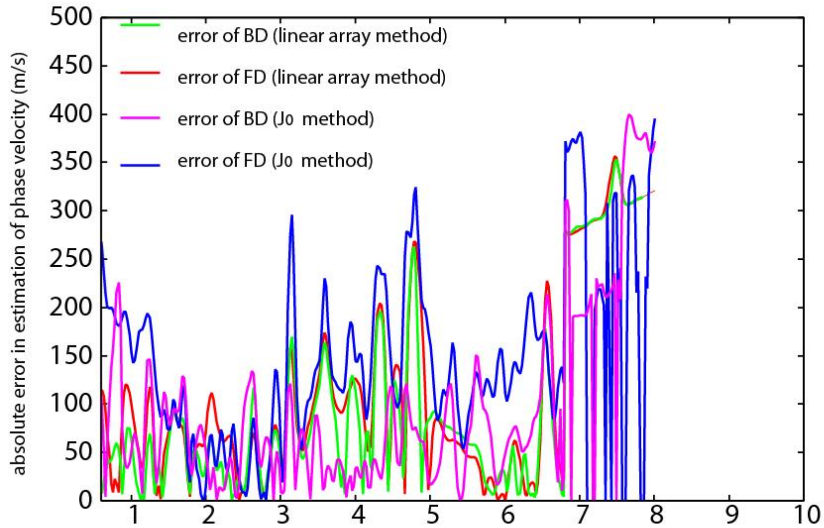

Fig. 13. The absolute difference between the estimated phase velocity and the theoretical velocity in each estimation

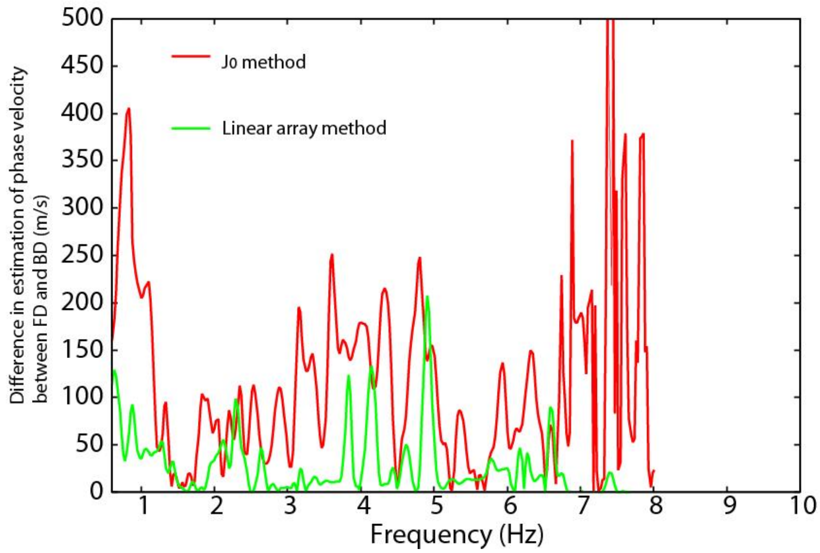

Fig. 14. The absolute difference between the estimated phase velocity from BD and FD in proposed method and J_0 method

For the SPAC method, it is observed that the estimation matches the theoretical one well in the frequency range 0.6 to $5.0Hz$. For the proposed method, the estimation matches well in the frequencyrange of 1.0 to $5.0Hz$ for the linear array (Fig. 11). In the kr_{max} range, it is 0.5to2.3, respectively.Let us go back to see the estimations in the numerical simulation in the last chapter (Figs. 8 and 9). We can see that in numerical simulation, the estimation has larger accurate range (kr_{max} range of 0.8 to 3.0) especially in high frequency range. One reason to cause the difference is the difference of wave field. In the field test, the wave field cannot be perfectly stationary or as simple as that in the numerical simulation. The uncorrelated noise or the coherent noise (not caused by plane Rayleigh wave) in high frequency range also may have bad influence on the result. As shown in Figs. 11 and 13, in high frequency range, there is quite large fluctuation. It may comes from the defect of the inversion technique. The influence caused by the uncorrelated noise and the defect of inversion technique would be discussed in the future more detailedly.

However, we can see that the estimation from the two linear arrays coincides with each other, which demonstrates the robustness of the proposed method against the azimuth. On the other hand, the J_0 method behaves quite differently according to the azimuth (Fig. 12). This difference between two methods is obvious by seeing Fig. 14 and Table 2.The accuracy of J_0 method is also worse than the proposed method (Fig. 13 and Table 1).

Though it seems that one of the two estimations using J_0 method coincides with the theoretical one, the lack of robustness makes it difficult to be available in real cases. The proposed method is not available in any wave field as discussed in the last section. However, because of taking J_2 and J_4 terms into consideration, it can be applied with more stability and reliability than J_0 method inreal cases. This is confirmed at least by this field test.

5. CONCLUSION

In the present study, we have proposed a new method for estimating the phase velocity of Rayleigh waves. Using the discrete formula of CCF, we can use a linear array to do the estimation. This can be applied to many urban areas where the circular arrays are difficult to set. There are several conclusions drawn below.

a. In case of extremely azimuth-dependent wave field (uni-directional wave), the proposed method is available when the angle between the wave and linear array is at least smaller than $60°$. Compared to J_0 method, the proposed method has wider applicablerange.
b. In case of moderately azimuth-dependent wave field, only one linear array is enough to obtain accurate simulation. Compared to J_0 method, the proposed method has wider applicablerange.
c. Through the field test, the proposed method can obtain the same accurate estimations as SPAC method and is stable regardless of azimuth while the J_0 method doesnot work properly.

ACKNOWLEDGEMENTS

This study was partially supported by Grant-in-Aid for Young Scientists (S) (No.21671003) of the Japan Society for the Promotion of Science (JSPS).

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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