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New Iterative Method for Solving the Fornberg-Whitham Equation and Comparison with Homotopy Perturbation Transform Method

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Abstract

This paper presents an approximate analytical solution of the nonlinear Fornberg-Whitham equation using the new iterative method (NIM). A comparison is made between the NIM results, homotopy perturbation transform method (HPTM) and the Adomian's decomposition method (ADM). The solution procedure reveals that NIM is a reliable, simple and effective. The proposed technique solves nonlinear problems without using Adomian's polynomials and He's polynomials which is a clear advantage of it over the decomposition method. The results reveal that the proposed algorithm is very efficient, simple and can be applied to other nonlinear problems.

Keywords: New iterative method; Homotopy perturbation transform method; Nonlinear Fornberg-Whitham equation.

1 Introduction

The Fornberg-Whitham equation [1] given as

$$u_t - u_{xxt} + u_x = uu_{xxx} - uu_x + 3u_x u_{xx}, \tag{1.1}$$

has a type of traveling wave solution called a kink-like wave solution and anti kink-like wave solutions. Such kinds of traveling wave solutions have never been found for the Fornberg-Whitham equation. Eq. (1.1) was used to study the qualitative behaviour of wave-breaking [2,3].

Many important phenomena occurring in various fields of engineering and science are frequently modeled through linear and nonlinear differential equations. However, it is still very difficult to obtain closed-form solutions for most models of real-life problems. A broad class of analytical methods and numerical methods were used to handle such problems.

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In recent years, various methods have been proposed such as homotopy perturbation method [4,11] finite difference method [12,13] Adomian decomposition method [14–19], variational iteration method [20–23], weighted finite difference techniques [24], Laplace decomposition method [25], but all these methods have some limitations. It is worth mentioning that the homotopy perturbation method is applied without any discretization, restrictive assumption or transformation and is free from round off errors. The Laplace transform is totally incapable of handling nonlinear equations because of the difficulties that are caused by the nonlinear terms.

In the present paper, we use both the new iterative method, proposed first by Gejji and Jafari [26] and homotopy perturbation transform method, proposed by Madani at el. [27], Khan and Wu [28]. The first method has proven useful for solving a variety of nonlinear equations such as algebraic equations, integral equations, ordinary and partial differential equations of integer and fractional order and systems of equations as well. New iterative method is simple to understand and easy to implement using computer packages and yield better results than the existing Adomain decomposition method [14], homotopy perturbation method [4] and variational iteration method [20]. The second method is an elegant combination of the Laplace transformation, the homotopy perturbation method, and He's polynomials. The proposed algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact solutions for linear and nonlinear partial differential equations.

The motivated by the ongoing research in this area, we use the new iterative method and homotopy perturbation transform method in solving the Fornberg-Whitham equation.

2 Basic Idea of NIM

To describe the idea of the NIM, consider the following general functional equation [26-30]:

$$u(x) = f(x) + N(u(x)), \tag{2.1}$$

where N is a nonlinear operator from a Banach space B→B and f is a known function. We are looking for a solution u of (2.1) having the series form

$$u(x) = \sum_{i=0}^{\infty} u_i(x). \tag{2.2}$$

The nonlinear operator N can be decomposed as follows

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}. \tag{2.3}$$

From Eqs. (2.2) and (2.3), Eq. (2.1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\left\{ N\left(\sum_{j=0}^{\infty} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}. \tag{2.4}$$

We define the recurrence relation:

$$u_0 = f, \tag{2.5a}$$

$$u_1 = N(u_0), \tag{2.5b}$$

$$u_{n+1} = N(u_0 + u_1 + \ldots + u_n) - N(u_0 + u_1 + \ldots + u_{n-1}), \qquad n = 1,2,3,\ldots . \tag{2.5c}$$

Then:

$$(u_0 + u_1 + \ldots + u_{n+1}) = N(u_0 + u_1 + \ldots + u_n), \qquad n = 1,2,3,\ldots ,$$

$$u = \sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right). \tag{2.6}$$

If N is a contraction, i.e.

$$\|N(x) - N(y)\| \le k\|x - y\|, \qquad 0 < k < 1,$$

then:

$$\|u_{n+1}\| = \|N(u_0 + u_1 + \ldots + u_n) - N(u_0 + u_1 + \ldots + u_{n-1})\| \tag{2.7}$$

$$\le k\|u_n\| \le \ldots \le k^n \|u_0\| \quad n = 0,1,2,\ldots,$$

and the series $\sum_{i=0}^{\infty} u_i$ absolutely and uniformly converges to a solution of (2.1) [31], which is unique, in view of the Banach fixed point theorem [32]. The k-term approximate solution of (2.1) and (2.2) is given by $\sum_{i=0}^{k-1} u_i$.

2.1 Reliable Algorithm

After the above presentation of the NIM, we introduce a reliable algorithm for solving nonlinear partial differential equations using the NIM. Consider the following nonlinear partial differential equation of arbitrary order:

$$D_t^n u(x,t) = A(u,\partial u) + B(x,t), \qquad n \in N, \tag{2.8a}$$

with the initial conditions

$$\frac{\partial^m}{\partial t^m} u(x,0) = h_m(x), \quad m = 0,1,2,\dots,n-1, \tag{2.8b}$$

where A is a nonlinear function of u and ∂ u (partial derivatives of u with respect to x and t) and B is the source function. In view of the integral operators, the initial value problem (2.8a) and (2.8b) is equivalent to the following integral equation

$$u(x,t) - \sum_{m=0}^{n-1} h_m(x)\frac{t^m}{m!} + I_t^n B(x,t) + I_t^n A = f + N(u), \tag{2.9}$$

Where

$$f = \sum_{m=0}^{n-1} h_m(x)\frac{t^m}{m!} + I_t^n B(x,t), \tag{2.10}$$

and

$$N(u) = I_t^n A, \tag{2.11}$$

where I_t^n t is an integral operator of n fold. We get the solution of (2.9) by employing the algorithm (2.5).

3 Basic Idea of Homotopy Perturbation Method (HPM)

Consider the following nonlinear differential equation [3-10]:

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{3.1}$$

with the boundary conditions of

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma,$$

where A, B, f(r) and r are a general differential operator, a boundary operator, a known analytic function and the boundary of the domain Ω, respectively. The operator A can generally be divided into a linear part *L* and a nonlinear part *N*. Equation (3.1) may therefore be written as.

$$L(u) + N(u) - f(r) = 0, \tag{3.2}$$

By the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0,1] \rightarrow R$ which satisfies

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{3.3}$$

or

$$H(v, p) = L(u) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{3.4}$$

where $p \in [0,1]$ is an embedding parameter, while u_0 is an initial approximation of (3.1), which satisfies the boundary conditions. Obviously, from (3.3) and (3.4) we will have

$$\begin{aligned} H(v,0) &= L(v) - L(u_0) = 0, \\ H(v,1) &= A(v) - f(r) = 0, \end{aligned} \tag{3.5}$$

The changing process of p from zero to unity is just that of $v(r, p)$ from u_0 to $u(r)$. In topology, this is called deformation, while $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopy. If the embedding parameter p is considered as a small parameter, applying the classical perturbation technique, we can assume that the solution of (3.3) and (3.4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots\infty. \tag{3.6}$$

Setting p = 1 in (2.6), we have

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \ldots. \tag{3.7}$$

The combination of the perturbation method and the homotopy method is called the HPM, which eliminates the drawbacks of the traditional perturbation methods while keeping all its advantages. The series (3.7) is convergent for most cases. However, the convergent rate depends on the nonlinear operator $A(v)$ .Moreover, He [3], made the following suggestions.

(1) The second derivative of $N(v)$ with respect to v must be small because the parameter may be relatively large; that is, $p \rightarrow 1$.

(2) The norm of $L^{-1}(\frac{\partial N}{\partial v})$ must be smaller than one so that the series converges.

4 Basic Idea of HPTM

To illustrate the basic idea of this method, we consider a general nonlinear non-homogeneous partial differential equation with initial conditions of the form [33,34]

$$Du(x,t)+Ru(x,t)+Nu(x.t)=g(x,t), \tag{4.1}$$

with the boundary conditions of

$$u(x,0)=h(x), \qquad u_t(x,0)=f(x),$$

where D is the second order linear differential operator $D=\dfrac{\partial^2}{\partial t^2}$, R is the linear differential

Operator of less order than D, N represents the general non-linear differential operator and g(x, t) is the source term. Taking the Laplace transform (denoted throughout this paper by L) on both sides of Eq. (4.1):

$$L[Du(x,t)]+L[Ru(x,t)]+L[Nu(x,t)]=L[g(x,t)]. \tag{4.2}$$

Using the differentiation property of the Laplace transform, we have

$$L[u(x,t)]=\frac{h(x)}{s}+\frac{f(x)}{s^2}-\frac{1}{s^2}L[Ru(x,t)]+\frac{1}{s^2}L[g(x,t)]-\frac{1}{s^2}L[Nu(x,t)]. \tag{4.3}$$

Operating with the Laplace inverse on both sides of Eq. (4.3) gives

$$u(x,t)=G(x,t)-L^{-1}\left[\frac{1}{s^2}L\big[Ru(x,t)+-Nu(x,t)\big]\right]. \tag{4.4}$$

where G(x, t) represents the term arising from the source term and the prescribed initial conditions. Now, we apply the homotopy perturbation method

$$u(x,t)=\sum_{n=0}^{\infty}p^n u_n(x,t), \tag{4.5}$$

and the nonlinear term can be decomposed as

$$Nu(x,t)=\sum_{n=0}^{\infty}p^n H_n(x,t), \tag{4.6}$$

for some He's polynomials H_n (see [35,36]) that are given by

$$H_n(u_0, u_1, u_2, \ldots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i u_i \right)\right]_{p=0}, \qquad n = 0,1,2,3\ldots.$$

Substituting Eqs. (4.6) and (4.5) in Eq. (4.4), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - p\left(L^{-1}\left[\frac{1}{s^2} L\left[R\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(x,t)\right]\right]\right), \quad (4.7)$$

which is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of p, the following approximations are obtained

$$p^0 : u_0(x,t) = G(x,t),$$
$$p^1 : u_1(x,t) = L^{-1}\left[\frac{1}{s^2} L\left[Ru_0(x,t) + H_0(u)\right]\right],$$
$$p^2 : u_2(x,t) = L^{-1}\left[\frac{1}{s^2} L\left[Ru_1(x,t) + H_1(u)\right]\right], \quad (4.8)$$
$$p^3 : u_3(x,t) = L^{-1}\left[\frac{1}{s^2} L\left[Ru_2(x,t) + H_2(u)\right]\right],$$

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5 Numerical Applications

In this section, we apply NIM and HPTM to solve the nonlinear Fornberg-Whitham equation. Consider the following nonlinear Fornberg-Whitham equation [37]:

$$u_t - u_{xxt} + u_x = uu_{xxx} - uu_x + 3u_x u_{xx}, \quad (5.1a)$$

with the initial condition

$$u(x,0) = e^{\frac{x}{2}}. \quad (5.1b)$$

Then, the exact solution is given by:

$$u(x,t) = e^{\frac{x}{2} - \frac{2t}{3}}$$

By NIM:

From (2.5a) and (2.10) we obtain

$$u_0(x,t) = e^{\frac{x}{2}},$$

Therefore, from (2.9), the initial value problem (5.1) is equivalent to the following integral equations:

$$u(x,t) = e^{\frac{x}{2}} + I_t\left(uu_{xxx} - uu_x + 3u_x u_{xx} + u_{xxt} - u_x\right). \tag{5.2}$$

Taking

$$N(u) = I_t\left(uu_{xxx} - uu_x + 3u_x u_{xx} + u_{xxt} - u_x\right)$$

Therefore, from (2.5), we can obtain easily the following .first few components of the new iterative solution for the equation (5.1):

$$\begin{aligned}
u_0(x,t) &= e^{\frac{x}{2}}, \\
u_1(x,t) &= \frac{-t}{2} e^{\frac{x}{2}}, \\
u_2(x,t) &= \frac{1}{8} e^{\frac{x}{2}}\left[-t + t^2\right], \\
u_3(x,t) &= -\frac{1}{96} e^{\frac{x}{2}}\left[3t - 6t^2 + 2t^3\right], \\
u_4(x,t) &= \frac{1}{384} e^{\frac{x}{2}}\left[-3t + 9t^2 - 6t^3 + t^4\right], \\
&\;\;\vdots
\end{aligned} \tag{5.3}$$

and the rest of the components of iteration formula (4.7) are obtained. The approximate solution which involves few terms is given by

$$u(x,t) = \sum_{i=0}^{4} u_i = \frac{1}{384} e^{\frac{x}{2}}\left(384 - 225t + 81\,\mathrm{t}^2 - 14\,\mathrm{t}^3 + t^4\right). \tag{5.4}$$

Table 1. The absolute errors for differences between the exact solution and 5th-order NIM, when t=5

x_i	u_{exact}	u_{NIM}	$\lvert u_{exact} - u_{NIM} \rvert$
-4	0.0048279499	0.0031719207	1.65602E-3
-2	0.0131237287	0.0086221743	4.50155E-3
0	0.0356739933	0.0234374990	1.22364E-2
2	0.0969719679	0.0637097231	3.32622E-2
4	0.2635971382	0.1731809521	9.04161E-2

By HPTM:

By applying the aforesaid method subject to the initial condition, we have

$$u(x,t) = \frac{e^{\frac{x}{2}}}{s} + \frac{1}{s} L\left[uu_{xxx} - uu_x + 3u_x u_{xx} + u_{xxt} - u_x\right], \tag{5.5}$$

The inverse of the Laplace transform implies that

$$u(x,t) = e^{\frac{x}{2}} + L^{-1}\left[\frac{1}{s} L\left[uu_{xxx} - uu_x + 3u_x u_{xx} + u_{xxt} - u_x\right]\right]. \tag{5.6}$$

Now, we apply the HPM

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = e^{\frac{x}{2}} + L^{-1}\left[\frac{1}{s} L\left[\sum_{n=0}^{\infty} p^n H_n(u) - \sum_{n=0}^{\infty} p^n H'_n(u) + 3\sum_{n=0}^{\infty} p^n H''_n(u) + (u_n)_{xxt} - (u_n)_x\right]\right]. \tag{5.7}$$

where $H_n(u), H'_n(u), H''_n(u)$ are He's polynomials [35, 36] that represents the nonlinear terms. So, the He's polynomials are given by

$$\sum_{n=0}^{\infty} p^n H_n(u) = uu_{xxx}.$$

The first few, components of He's polynomials, are given by

$$\begin{aligned}
&H_0(u) = u_0 (u_0)_{xxx}, \\
&H_1(u) = u_0 (u_1)_{xxx} + u_1 (u_0)_{xxx}, \\
&H_2(u) = u_0 (u_2)_{xxx} + u_1 (u_1)_{xxx} + u_2 (u_0)_{xxx}, \\
&. \\
&. \\
&.
\end{aligned} \tag{5.8}$$

for $H'_n(u)$ we find that

$$\sum_{n=0}^{\infty} p^n H'_n(u) = uu_x,$$

$$H'_o(u) = u_0(u_0)_x, \tag{5.9}$$

$$H'_1(u) = u_0(u_1)_x + u_1(u_0)_x,$$

$$H'_2(u) = u_0(u_2)_x + u_1(u_1)_x + u_2(u_0)_x,$$

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and for $H''_n(u)$, we find that

$$\sum_{n=0}^{\infty} p^n H''_n(u) = u_x u_{xx},$$

$$H''_0(u) = (u_0)_x(u_0)_{xx}, \tag{5.10}$$

$$H''_1(u) = (u_0)_x(u_1)_{xx} + (u_1)_x(u_0)_{xx},$$

$$H''_2(u) = (u_0)_x(u_2)_{xx} + (u_1)_x(u_1)_{xx} + (u_2)_x(u_0)_{xx},$$

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Comparing the coefficients of like powers of p, we have

$$p^0 : u_0(x,t) = e^{\frac{x}{2}},$$

$$p^1 : u_1(x,t) = L^{-1}\left[\frac{1}{s} L\left[H_0(u) - H'_0(u) + 3H''_0(u) + (u_0)_{xxt} - (u_0)_x\right]\right] = \frac{-t}{2} e^{\frac{x}{2}},$$

$$p^2 : u_2(x,t) = L^{-1}\left[\frac{1}{s} L\left[H_1(u) - H'_1(u) + 3H''_1(u) + (u_1)_{xxt} - (u_1)_x\right]\right] = e^{\frac{x}{2}}\left[\frac{-t}{8} + \frac{t^2}{8}\right], \tag{5.11}$$

$$p^3 : u_3(x,t) = L^{-1}\left[\frac{1}{s} L\left[H_2(u) - H'_2(u) + 3H''_2(u) + (u_2)_{xxt} - (u_2)_x\right]\right] = e^{\frac{x}{2}}\left[\frac{-3t}{96} + \frac{6t^2}{96} - \frac{2t^3}{96}\right],$$

$$p^4 : u_4(x,t) = L^{-1}\left[\frac{1}{s} L\left[H_3(u) - H'_3(u) + 3H''_3(u) + (u_3)_{xxt} - (u_3)_x\right]\right] = e^{\frac{x}{2}}\left[\frac{-3t}{384} + \frac{9t^2}{384} - \frac{6t^3}{384} + \frac{t^4}{384}\right],$$

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and the rest of the components of iteration formula (4.7) are obtained. The approximate solution which involves few terms is given by

$$u(x,t)=\sum_{i=0}^{4} u_i = e^{\frac{x}{2}}\left(1-\frac{85}{128}t+\frac{27}{128}t^2-\frac{7}{192}t^3+\frac{1}{384}t^4\right). \tag{5.12}$$

Table 2. The absolute errors for differences between the exact solution and 5th-order HPTM when t=5

x_i	u_{exact}	u_{HPTM}	$\lvert u_{exact}-u_{HPTM}\rvert$
-4	0.0048279499	0.0031719207	1.65602E-3
-2	0.0131237287	0.0086221743	4.50155E-3
0	0.0356739933	0.0234374990	1.22364E-2
2	0.0969719679	0.0637097231	3.32622E-2
4	0.2635971382	0.1731809521	9.04161E-2

which are the absolute errors in Table 1 by NIM the same results obtained by HPTM in Table 2 and ADM [37]. The obtained results prove that the NIM described method is very simple and easy method compared with the other methods and give the approximate solution in series form, this series in closed form gives the corresponding exact solution of the given problem.

In Fig.1 the approximate solutions obtained by the 7-order of NIM and HPTM have been plotted, It is very remarkable to see that the surfaces of the two approximate solutions are in high agreement with the surface of the exact solution

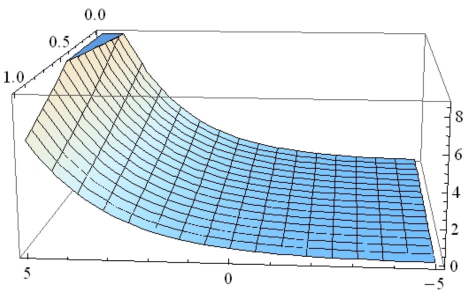
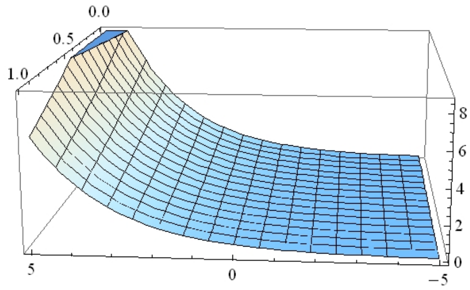


Fig. 1.a



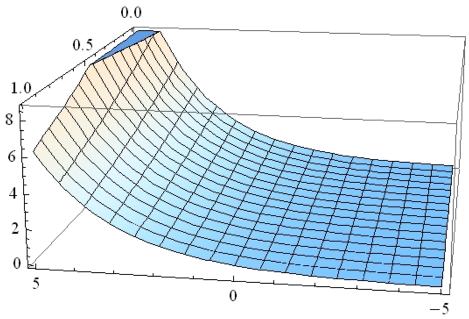


Fig. 1.b

Fig. 1.c

Fig. 1. The behavior of the solutions obtained by (Fig.1.a) NIM, (Fig. 1.b) HPTM, (Fig. 1c) Exact solution, with different values of x, t, for $x:-5\to 5,\ t:0\to 1$.

In Fig. 2 the approximate solutions obtained by the 7-order of NIM and HPTM have been plotted, which coincide with the exact solution.

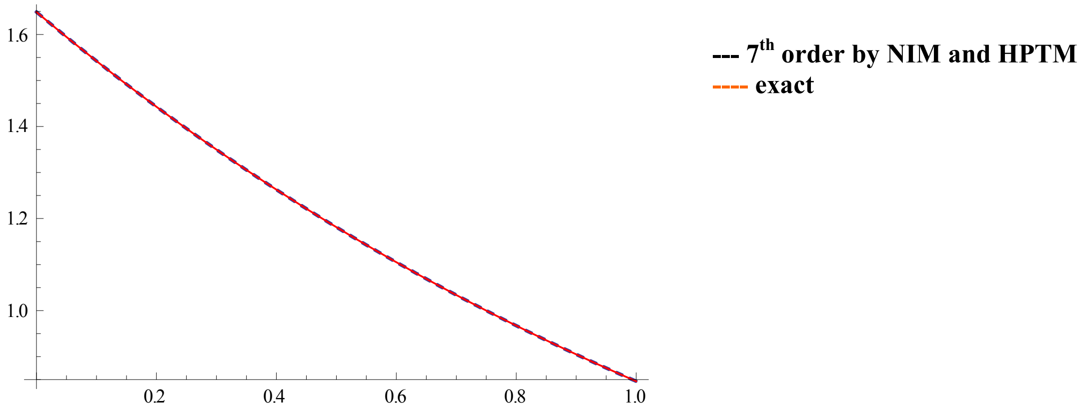


Fig. 2. Comparison between approximate solution by NIM, HPTM and Exact solution of $u(x,t)$ in case $x=1,\ t:0\to 1$

6 Conclusions

In this paper, the NIM and HPTM were successfully applied for finding the approximate solutions of the nonlinear Fornberg-Whitham equation with initial conditions. The fact that the HPTM solve nonlinear problems without using Adomian's polynomials but the NIM solve nonlinear problems without using Adomian's polynomials and He's polynomials is a clear advantage of this technique over the decomposition method. The results show that the two methods are powerful and efficient techniques in finding exact and approximate solutions for nonlinear differential equations.

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Competing Interests

Authors have declared that no competing interests exist.

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