Asian Journal of Education and Social Studies

Volume 47, Issue 1, Page 1-14, 2023; Article no.AJESS.102185
ISSN: 2581-6268

Investigating Misconceptions in Linear Equations and its Impact on Students' Achievement at Middle Secondary Schools of Bhutan

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJESS/2023/v47i11012

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/102185

Original Research Article

Received: 24/04/2023
Accepted: 27/06/2023
Published: 13/07/2023

ABSTRACT

Aims: The objective of this study was to investigate and address ninth-grade students' misconceptions in learning linear equations with the use of three-tier diagnostic test.

Study Design: The present study is built upon an explanatory sequential mixed approach, in which the researcher first does quantitative research, evaluates the results, and then uses the results to create a more in-depth qualitative analysis. The qualitative data is viewed as explanatory because it helps to explain the initial quantitative results.

Methodology: This study used a mixed-method approach, gathering, analyzing, and interpreting both quantitative and qualitative research in a single study. This strategy was chosen since the study's objective was to thoroughly analyze students' misconceptions about learning linear equations using both a quantitative and a qualitative approach.

Results: The conceptual knowledge examined in the test items were balancing equation, equation with brackets, fractions and negative signs, finding the table of values, finding slope and y-intercept, and graphing the equations. The diagnostic test was administered to 29 ninth grade students. It

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was revealed that majority (58.6%) of the students possessed misconceptions. The sources of misconceptions were attributed to inadequate mastery of subject knowledge of teachers, wrong information in the textbooks, prior knowledge and imprecise use of terminology.

Conclusion: Mobile apps were used to rectify the misconceptions held by the students to prevent the propagation of those misconceptions in students. Thus, a teacher can successfully read the cognitive conflict of his or her pupils and increase the likelihood of conceptual change by identifying, interpreting, and controlling it. A teacher can also successfully assist and lead students to engage in meaningful mathematics learning experiences.

Keywords: Misconceptions; investigation; linear equation; conceptual knowledge; students' achievement.

1. INTRODUCTION

Misconceptions as a conceptual obstacle are hindrance to learning mathematics. It is anticipated that students will actively participate in building their mathematical knowledge through the dissemination and fusion of new ideas. Therefore, until the misconceptions are identified and rectified, there is a danger of cognitive conflict and more errors when there are misconceptions. Similar to this, Mulungye, et al. [1] research showed that misconceptions about mathematics are a recurring theme among all cohorts. Even students who are good at mathematics will consider mathematics as one of the most difficult subjects in the school curriculum if corrective action is delayed. Among all the branches of mathematics, algebra also can be a way to address the issue of declining performance in Mathematics. Several studies have shown that introductory algebra is essential for developing students` knowledge of mathematics when they experiment with abstract ideas that form the foundation of algebra.

There might be many factors that impact learning in mathematics, but one factor that impedes learning is a misconception. One of the most significant barriers to learning mathematics has been identified as misconceptions. Misconception can be observed at every age and every educational level. Furthermore, Ojose [2] claimed that it is well-known that students of all grade levels have misunderstandings about a variety of mathematical subjects. In a similar vein, misconceptions about mathematical concepts and a lack of understanding of the core ideas prevent students from learning the concepts when they are presented with new mathematical performance challenges. It is critical to identify and eliminate misunderstandings in mathematics because, according to Ay [3], it is required to have a working knowledge of earlier concepts in order to learn new ones.

1.1 Statement of the Problem

Most educators agree that algebra is a fundamental and important component of mathematics teaching and learning and is regarded as the core topic, particularly for secondary school mathematics. Researchers in the domains of mathematics education and educational psychology have identified a variety of misconceptions that students have about algebra throughout the course of the last few decades [4]. Algebra is an expanded form of operations that students do on fractions and integers that involve symbols and algebraic ideas. As a result, misconceptions about fractions have a big impact on how well students learn mathematics. Although they are aware that algebra involves variables, many students find algebra to be difficult to comprehend and difficult to learn.

Therefore, addressing the misconceptions in algebra in grade nine can enhance their academic success in mathematics, ultimately addressing the poor performance of Bhutanese students in Mathematics. The performance of Bhutanese students in Mathematics is declining every year as revealed in Pupil's Performance Report of Bhutan Council for School Examinations and Assessment (Bhutan Council for School Examinations and Assessment [BCSEA] [5,6,7] and the mean marks in the last three years have declined, as 56.83%, 51.49% and 46.80% respectively. According to Mulungye et al. [1] most teachers make little use of students' mathematics ideas which is called student-based methods than subject-based methods. They also point out that the main difficulty seems to be in the ability of teachers to use their knowledge of students' mistakes rather than in their awareness of mistakes. It resulted in teaching strategies that did not address student difficulties and deficits in algebra. Moreover, teachers not only need assistance in identifying misconceptions but also need to identify how to address the misconceptions in the whole process of teaching and learning mathematics. These are assumed solutions to the above-stated problems. Thus, problems of this nature are essential to investigate.

1.2 Objectives

The main aim of this study is to investigate misconceptions about grade nine students in linear equations. Further, it investigates its impacts on the student`s performance in mathematics.

More specifically, this study aimed to achieve the following specific objectives:

- Identify the types of students' misconceptions in solving linear equations.
- Identify the causes of the misconception concerning the operations involved in linear equations.
- Suggest key measures to overcome the misconception.

1.3 Research Questions

What are misconceptions in learning Linear equations faced by students of Trashigang middle secondary school, and how can these be addressed?

1.3.1 Sub questions

1. What are the misconceptions in linear equations faced by class IX students?
2. What are the causes of misconceptions in the linear equation?
3. What is the impact of students' misconceptions on their learning achievement in the linear equation?
4. What are the advantages of addressing misconceptions?

2. LITERATURE REVIEW

2.1 Definition

A misconception is a misunderstanding of the concepts which is embedded in students' mind and a misconception is a drawing of views or opinions that is incorrect based on faulty thinking. Similar to this, misconceptions can be described as incorrectly interpreting concepts based on naive theories that obstruct learners' ability to reason critically. Further, the misconception is mismatching of the scientific definition of concept and definition formed in students' minds and it conflicts with the expert opinions in related areas [8].

2.2 Types of Misconceptions

Misconceptions in educational studies widely started during 1980s and they are still persistent [9]. According to Demirci [10] misconceptions are grouped into five categories such as preconceived notions, non-scientific beliefs, conceptual understanding, vernacular misconceptions and factual misconceptions. Five categories of misconceptions are described as follows; preconceived notions are information that is already encrypted before somebody teaches the concept, non-scientific are those learned from a religious perspective, vernacular misconception pertains to the usage of words, conceptual misunderstanding arises when one cannot connect to new information and factual misconceptions are falsities learned at the early age and retained unchallenged into adulthood.

Algebra is frequently seen as the key to further education and related professional routes in mathematics. Although algebraic understanding is considered crucial for student success in more advanced mathematics courses, many scholars have documented that students struggle with algebraic concepts, especially those relating to linear equations. "Algebraic misconceptions were classified into four categories. These categories are algebraic expressions, linear equations, polynomials, exponents and radical expressions, and finally functions and graphs" [11 p. 805]. Among four major misconceptions of algebra, this study will focus on the misconceptions in linear equations.

2.3 Causes of Misconceptions in Linear Equations

According to Butuner and Baki [12], there are three phases in mathematics. These are referred to as the verbal period, during which everything was recorded in writing, the abbreviation period, and the symbol period. According to the studies, students have trouble learning the algebraic period, also known as the symbolic period; Equations and the concept of the variable, which is the primary ingredient in equations, have both been thoroughly discussed in relation to algebra [13]. Essentially, the reason people have negative prejudices about math and show no effort to learn is that they perceive math lessons as a lesson made entirely of abstract concepts.

Thus these factors can also contribute to students having misconceptions in learning certain concepts in Mathematics.

Khalid and Embong [14] mentioned a number of reasons for this problem, one of which is that students are taught to follow rules and procedures in a very abstract way without using models to help them understand the concepts. The teachers' hasty completion of the extensive curriculum was probably the cause of this. The main causes of errors were found to be carelessness, inadequate fundamental knowledge, such as the inability to multiply and divide even whole numbers, an inability to integrate concepts of integers because they are accustomed to whole number schema, and rule mix-ups, which are also the result of superficial understanding.

2.4 Advantages of Addressing Misconceptions

A better knowledge of processes and the caliber of learning that would happen will result from identifying students' algebraic challenges and misconceptions. According to Chow [15], teachers will have a better understanding of the subject matter, teaching methods, and students' learning processes if they are aware of the misunderstandings that students have about algebra. in order to produce worthwhile opportunities to improve learning. Since misconceptions are the biggest obstacle to learning, it is crucial to overcome them in order to improve learning.

According to Mulungye et al. [1], learning and misconceptions in mathematics are a recurrent theme that goes through all cohorts, and if corrective interventions are not given on time, many capable children would view mathematics as a difficult subject in the school curriculum. Similarly, Das [16] claimed that teachers might create effective instructions to avoid misconceptions if research could identify students' misconceptions. Therefore, doing research on students' misunderstandings is a means to aid both teachers and students in their instruction and learning. It is crucial for teachers to recognize and correct any misunderstandings that exist among their students since they can hinder their performance. However, Woodward, et al. [17] pointed out that a persistent, superficial grasp of mathematics enables students to apply incorrectly.

2.5 Impacts of Misconceptions of Students' Achievement

Mathematical misunderstandings have a negative impact on student's achievement because they cause them to misunderstand instructions and give erroneous answers, both of which impedes their academic standing. According to Schnepper et al. [18], when specific mistakes committed by particular students were addressed with formative assessment data guiding the teaching to correct that misconception, it suggests that specific student achievement improved. As Riccomini [19] noted that the discovery and study of students' arithmetic errors have the potential to enhance instructional design and, ultimately, student performance.

Students cannot fully learn arithmetic if they do not comprehend the connections between algebraic and pictorial representations [20]. Students who have misconceptions regarding the equals sign or negative signs, for instance, answer fewer equations correctly and struggle more while learning how to do so [21]. Correcting these misunderstandings can help you become more adept at solving equations [21]. The introduction of equations is frequently hampered by this operational [22] or procedural [23] approach, in part because good equation solving is predicated on a relational [23] or structural [22]. In conclusion, students often have trouble solving equations because expressions like $3x + 1$ cannot be understood as objects that, in relational terms, are also subject to operations because of operational viewpoints on those expressions [24,25]. hence, algebraic misunderstandings impede the learning score in Mathematics.

2.6 Key Measures to Overcome Misconceptions

Misconceptions can take many different forms, so it is important to correct the mistake as soon as possible to stop it from happening again. If misconceptions are not corrected, students may have serious gaps in their understanding of mathematical topics that will persist into subsequent years. To quickly correct the misconceptions, remedial action is crucial. Explicit instruction is defined by problem-solving models, a variety of examples, practices combined with feedback, and students' verbalizations of their thought processes [26]. Additionally, the NMAP [26] suggests that students who struggle in math receive regular education in the subject.

Simulated experiential learning has been advocated as a way to dispel students' preconceptions [27]. Additionally, experience learning fosters higher-order learning, which supports analytical thinking skills and self-directed learning [28]. Furthermore, according to Hakeem [29], students who engage in experiential learning have a deeper comprehension of their subject area than those who attend lectures solely. According to Rocha [30], students who were enrolled in classes that used experiential learning approaches thought they were more capable practitioners than those who were in classes have not. Additionally, using mathematical software and mobile apps has a bigger impact on learning mathematics. The mobile apps GeoGebra, Photomath, and Cymath are frequently utilized.

3. METHODOLOGY

3.1 Research Design

Study designs, according to Creswell [31], are styles of inquiry that fall within qualitative, quantitative, and mixed methodologies approaches and give precise guidelines for study design procedures. The current study is built on an explanatory sequential mixed approach, in which the researcher first carried out quantitative research, then analyzed the results, and then drew on the results to provide a more in-depth qualitative analysis. Additionally, the qualitative data is considered explanatory because it clarifies the earlier quantitative results [32].

First pre-test for baseline data was analyzed and subsequently interviewed the students to confirm their misconceptions, followed by designing and implementing required interventions for two weeks. Secondly, collected post-test data, analyzed, and compare the result with the pre-test result. The researcher employed a semi-structured interview_to investigate the common misconceptions about learning linear equations in ninth grade. The interview was conducted to examine and confirm the misconceptions (MC). This design allowed the researcher to examine and understand the topic in far more detail through a collection of data in both quantitive and qualitative ways. Moreover, since the study intended to study the misconception of individual teachers and students involved in the one-school setup, the convergent mixed method was most appropriate since it allows for the collection of quantitative and qualitative data in a short amount of time

3.2 Participants

Sampling is the process of choosing individuals so that statistical conclusions about the characteristics of the entire population can be drawn from them [33]. One of the middle secondary schools under Trashigang Dzongkhag served as the site for this investigation. Purposeful sampling was used by the researcher to choose individuals. The purposive sampling approach is predicated on the notion that the researcher sought to learn, comprehend, and acquire insight and, as a result, needed to choose a sample from which data could be easily gathered [34]. As a result, the school was specifically chosen for this study based on convenience, location, need, and demand. The pre-test to identify common misunderstandings in the solution of linear equations and the post-test to assess the effects of the targeted MC were given to all 29 ninth-grade mathematics students.

3.3 Sample Size

For the quantitative study, the sampling frame was chosen based on convenient sampling whereby the researcher selected a particular middle secondary school with a total sample of 29 participants from ninth grade students.

Table 1. Population and sample size

Participants	Gender		Total
	Male	**Female**	
Students	9	20	29

In addition, 17 students were selected based on the mistake made in pre-test for semi-structured interview to collect qualitative data for confirmation of misconceptions. The questions for the semi-structured was (adopted from Basem, 2019) as per the misconceptions of students have in linear equation.

3.4 Research Instrument

The instrument of the study was one-group pre-test-post-test design. Pre-test and post-test was three-tier diagnostic test and interview on the misconceptions in solving linear equations algebraically. The types of misconceptions were collected from pre-test and further confirmed by interview. The impact of the intervention was collected by conducting post-test after intervention. This research employed the following instruments to collect data and conduct the study.

3.4.1 Mathematics Achievement Test (MAT) (Pre-test and post-test: Content-based-questions)

The test was on covered topics that were *Solving linear equations algebraically*. The uniform mathematics tests (pre-test) three-tier diagnostic test of five sets were administered to the whole class. The researcher has made a special effort to prepare the Mathematics Achievement Test (MAT) in consultation with the senior mathematics teacher and two lectures from Samtse College of Education. The questions consisted of fine content validity and generally met the requirement of the teaching objectives test on covered topics which was *Solving linear equations algebraically*.

3.5 Data Collection Procedures

The intervention continued for three weeks during normal class sessions and after class hours. There were five sessions per week, each session of 50 minutes. All students from ninth grade appeared a pre-test on the topic *solving linear equations algebraically*. The students with misconceptions were treated and taught employing intervention (remediation) for three weeks long and all the students appeared on the post-test to see the impact of the intervention.

3.6 Data Analysis

SPSS IBM version 22 and Microsoft Excel (office 2016) were used for descriptive analysis in a quantitative method and thematic analysis was done for the qualitative method. The score of the pre-test and post-test was compared by descriptive analysis (Comparing mean and median). Pre-test data were analyzed using Excel to find the possible misconceptions and types of misconceptions in linear equations. Further used Excel to compare the pre-test and post-test item analysis. The pre-test item analysis was conducted as stated in Table 2.

3.6.1 Thematic analysis of semi-structured interview

Thematic analysis was used for semi-structured interview. It was a good approach for researchers to try to find out something about people's views, opinions, knowledge, and experiences from a set of qualitative data such as interviews, transcripts, profiles and survey responses [32]. The researcher adopted six phases of thematic analysis by Clarke and Braun [35].

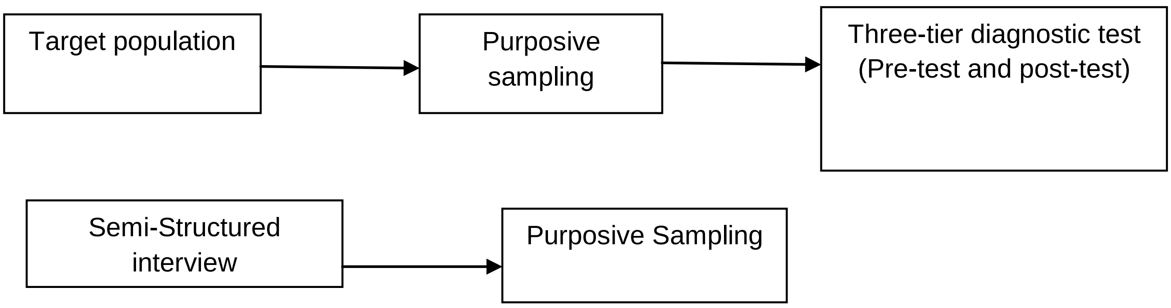

Fig. 1. Sampling procedure

Table 2. Possible misconceptions

	SET I			SET II			SET III			SET IV			SET V		
	a	**b**	**C**	**a**	**b**	**C**	**A**	**B**	**c**	**a**	**b**	**c**	**a**	**b**	**c**
Correct	24	26	12	14	2	7	11	3	3	1	0	1	3	2	1
percent	82.8	89.7	41.4	48.3	6.9	24.1	37.9	10.3	10.3	3.4	0.0	3.4	10.3	6.9	3.4
Wrong	4	3	15	11	16	16	13	18	19	21	23	17	17	17	11
percent	13.8	10.3	51.7	37.9	55.2	55.2	44.8	62.1	65.5	72.4	79.3	58.6	58.6	58.6	37.9
Blank	1	0	2	4	11	6	5	8	7	7	6	11	9	10	17
percent	3.4	0.0	6.9	13.8	37.9	20.7	17.2	27.6	24.1	24.1	20.7	37.9	31.0	34.5	58.6
Total	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0

Table 3. Confirmed misconceptions

Areas of misconception	Balancing equations	Negative sign, brackets and fractions	Table of values	Slope and Y-intercepts	Graphing
No. of students	2	12	15	17	16
Percent	6.9	41.4	51.7	58.6	55.2

3.7 Data Triangulation

Clinical researchers employ four types of triangulation such as method triangulation investigator triangulation, theory triangulation and data source triangulation [36,37,38]. In this particular study method triangulation was employed. According to Moon [39] method triangulation incorporates collecting data from different research methods. In this study, pre-test, post-test and semi-structured interviews were triangulated for the validation of results as it offers a variety of datasets to explain different aspects of the phenomenon of interest. Further, the pre-test was supplemented by face-to-face semi-structured interviews and supported by the post-test achievement. Thus, the triangulation of the research results gave more confidence in the research findings [40].

3.8 Reliability and Validity

To determine the validity of the questions, two specialists (two lecturers of Samtse College of Education and one experienced mathematics teacher) were involved. Language appropriateness, clarity, content coverage and content relevancy of the pre-test questions and interview questions were rated and rectified by the three experts. Moreover, the credibility of the study is also important for the study. The study's credibility builds confidence in the validity of the research findings [14]. In this study, a panel of experienced researchers provided feedback, which was incorporated into the project. Further, the researcher had numerous briefing sessions by the lecturers and feedback on data and interpretation to obtain from the participants.

4. RESULTS

4.1 Demographic Profile of the Participants

The participants comprised 29 grade ninth students, of which 31.1% were male and 69.9% were female (Table 4). In addition, a total of seventeen participants were involved in a semi-structured interview to confirm their misconceptions.

4.2 The Possible Misconceptions

The following section answers the research question: What misconceptions in learning linear equations are faced by students? The following results in table 5 are the possible misconceptions of the sample students revealed through the pre-test. The pre-test was conducted mainly to look for errors/mistakes through which possible misconceptions have been identified that need to be confirmed.

The above result shows that 6.9% of the sample students have misconceptions in balancing the equation; 41.4% in variable and constant terms involving negative signs and brackets; 62.1% in filling the table of values; 93.1 % in graphing; and 96.6% in identifying slope and y-intercepts. It means that the majority (96.6%) of the sample students have misconceptions about identifying slope and y-intercepts, and the minority (6.9%) balances the equation.

4.3 Confirmed Misconceptions and Sources of Misconceptions

This section presents the result of the research question study: What are the factors that cause these misconceptions? The following results are the confirmed misconceptions, and sources of misconceptions were revealed through the semi-structured interview. The semi-structured interview was mainly conducted to confirm the misconceptions and find out the sources of misconceptions that are identified and confirmed. The thematic analysis was carried out for an interview, and the following are the result revealed by 17 sample students.

i. *Can you tell me how you solved the question? How did you get this?*

The sample student responded that they thought the “answer was correct and they followed the right steps” though the answer was wrong (S1, S2, S3, S4, S5, S12, S15). Some responded that they “got mixed up with an x-variable, negative sign, and brackets” (S6, S7, S8, S9, S10, S11, S16, S17).

ii. *Have you checked the correctness of the solution that you found for this problem? Why did you write like this?*

The sample student responded that *'No'*, they have "not checked the correctness" (S3, S4, S6, S7, S8, S10, S17) and they thought their "answer was correct" (S1, S2, S9, S11, S12, S13, S14, S15).

iii. *Can you check it now? When did you learn?*

Responded *'No'*, "do not know how to check the answer" (S1, S2, S3, S8, S9, S11) and responded that they have "learned in lower classes regarding linear equations" (S2, S3, S7, S12, S13, S14, S15, S17).

iv. *Have you understood why the answer you found is not true? Who taught you to solve in this way?*

Sample students responded that their "answer was wrong" (S4, S5, S6, S7, S9, S10, S11, S13, S17). Sample students revealed that they were taught by 'teachers' (S1, S2, S4, S5, S7, S9, S12, S13, S15, S17), taught by 'parents' (S13), taught by 'brother and friends' (S11, S8), and learned from 'textbook' (S15).

V. *What method you used in other problems? Where did you learn in this way?*

Students responded that they used the 'direct substitution method' (S2, S3, S5, S6, S9, S14, S16). One sample student responded, 'no method was used, simply used calculator' (S4).

The above interview results revealed misconceptions about solving linear equations in various conceptual understandings. It also revealed that teachers and textbooks are the leading causes of misconceptions, and few pointed out that they have learned from parents and friends.

The following results in Table 6 are the confirmed list of misconceptions of the sample students revealed through the pre-test and semi-structured interview. The semi-structured interview was mainly conducted to confirm the misconceptions further.

The above result shows the confirmed misconception with 2 students in balancing equations with 6.9%, 12 students in solving negative, brackets, and fractional problems with 41.4%, 15 students in finding the table of values with 51.7%, 17 students in finding slope and y-intercepts with 58.6% and 16 students in graphing the equation with 55.2%.

The above result revealed the confirmed misconceptions after conducting semi-structured interview. There were 2 students who had misconceptions in *balancing equation*, 12 students with *identifying slope and y-intercepts,* 15 students with *finding the table of values,* 17 students with *identifying slope and y-intercepts* and 16 students in *graphing* out of 29 (twenty-nine) sample students.

4.4 Impact of Misconceptions on Learning Achievement

This section presents the result of the research question study: What are the impacts of misconceptions on students' learning achievement in the linear equation? The following results are revealed while comparing the pre-test and post-test. The post-test was mainly conducted to check the impact of the intervention on their achievement. A similar set of questions were designed for the pre-test and post-test.

Table 4. Possible misconceptions through the pre-test

Areas possible of misconception	**Balancing equations**	**Negative sign, brackets and fractions**	**Table of values**	**Slope and Y-intercepts**	**Graphing**
No. of students	2 (6.9%)	12 (41.4%)	18 (62.1%)	28 (96.6%)	27 (93.1%)

Table 5. Confirmed misconceptions

Area of misconception	**Balancing equations**	**Negative sign, brackets and fractions**	**Table of values**	**Slope and Y-intercepts**	**Graphing**
No. of students	2	12	15	17	16
Percent	6.9	41.4	51.7	58.6	55.2

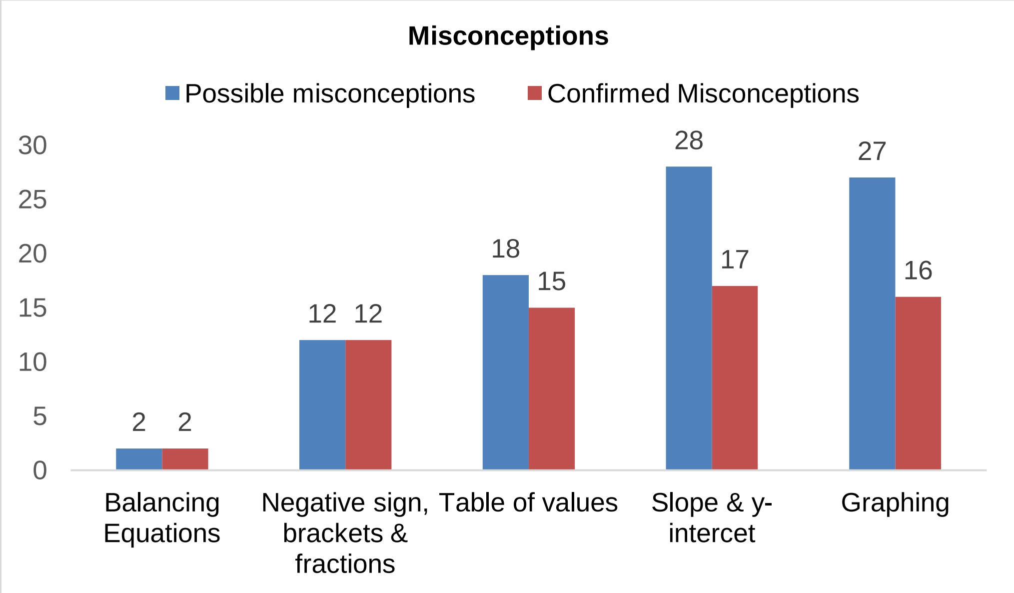

Fig. 2. Comparing possible misconceptions and confirmed misconceptions

The above result shows that 50.8% of the questions were solved correctly with an increment of 28.2% in the post-test compared to the pre-test. Wrong answer decreased to 45% from 50.8%, and the number of questions unattended was just 4.1% compared to 23.2% in the pre-test. It means that, after the intervention, some of the misconceptions were addressed and corrected. The correct answers significantly increased, the wrong answers reduced, and very few questions were kept blank.

The above graph shows the number of samples of students that have solved the questions incorrectly and had kept blank. In the pre-test, 7 students solved correctly, 15 students made mistakes, and 7 students were kept blank out of 29 sample students. In the post-test, 15 students solved correctly, 13 students made mistakes, and only 1 student kept blank. The above result indicates a significant improvement in post-test compared to pre-test from item analysis.

4.5 Advantages of Addressing Misconceptions

This section presents the study result of the research question: What are the advantages of addressing misconceptions? The result revealed that addressing misconceptions has many advantages for academic achievement. The independent sample t-test is conducted to compare the mean score of the pre-test and post-test.

A significance level of $P<0.05$, Cohen's d value: d=0.2-small effect, d=0.5-medium effect, d=0.8-large effect.

Table 8 shows that the mean difference between the pre-test and post-test was 7.38, and the two-tailed significance value (p) was 0.000, which was lower than the significant value $p<0.05$; this indicated a statistically significant difference in the post-test. That shows that the test scores in the post-test were significantly higher than the test scores of the pre-test.

Table 6. Comparing pre-test and post-test item analysis

	Test	**Percent (%)**
Correct	Pre-test	22.6
	Post-test	50.8
Wrong	Pre-test	50.8
	Post-test	45.0
Blank	Pre-test	23.2
	Post-test	4.1

The whole class was treated with intervention for three weeks using mobile apps (Cymath, Photomath, and GeoGebra software) to teach linear equations. The intervention impacted the learning achievement of students, which reveals that the standard deviation of the means of the pre-test was 5.58, and the standard deviation of the means of the post-test was 7.95. It indicated that the participants who were taught using intervention significantly reduced the level of misconceptions. With the intervention, misconceptions of some students were remediated.

5. DISCUSSION

5.1 Types of Misconceptions

The type of misconceptions as revealed from this study was *conceptual misunderstanding* of the mathematics content: balancing equations, fractional equations, identifying slope and y-intercepts and graphing the equations. Such type of misconceptions has been revealed from the result of the pre-test. The students also seem to have difficulties with algebraic operations which might be due to some types of prevailing misconceptions. The outcome is consistent with Setainingrum et al. [41] finding that certain children have trouble learning algebra because they have trouble distinguishing the variables, coefficients, and constants.

Further, the semi-structured interview question *how did you solve the problem?* Sample of students responded with an assumption that their answers were right although their answers were incorrect. Similarly, few responded that it was challenging with the bracket and negative signs while solving equations. This conclusion is supported by Toka and Asker [42] and Chow [15], who also discovered misconceptions about utilizing the distributive property wrongly, misinterpreting the rule that x signifies a different number, and failing to include literal symbols as variables.

Based on the points of above discussion, an assumption can be made that the common types of misconceptions of grade ninth students of the sample school are *conceptual misunderstanding, preconceived knowledge of algebra, and* misconceptions related to algebra are very common as evidenced by the literature. Further, the content analysis of the pre-test results and interview data indicated that the difficulties of students in dealing with the use of distributive property in algebraic operations can be due to incorrect *preconceived knowledge of algebra.* Thus there are two types of misconceptions: *conceptual misunderstanding and wrongly preconceived knowledge.*

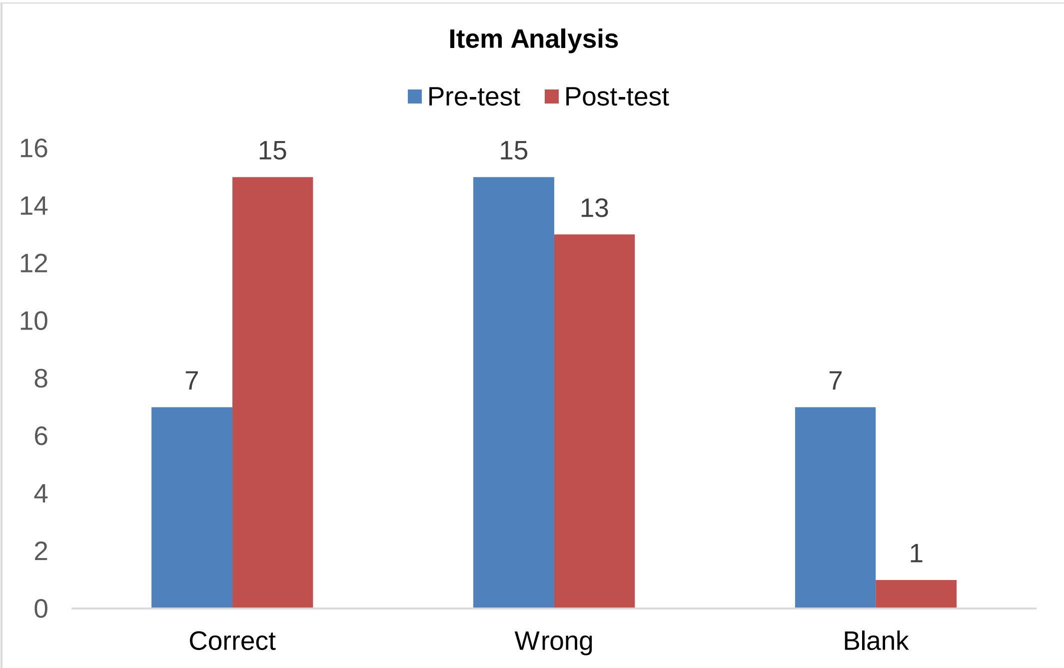

Fig. 3. Comparison between pre-test and post-test

Table 7. Independent sample t-test

Test	N	Mean	Mean Diff.	Std. deviation	Sig.(2 tailed)	Cohen's d
Pre-test	29	8.34	7.38	5.58	.000	3.11
Post test	29	15.72		7.95		

5.2 Causes of Misconceptions in Linear Equations

The leading causes of misconceptions could be teachers and textbooks. The findings were consistent with the need for teachers to explain to students that letters are not math symbols and that textbooks may contain errors, misleading or contradictory representations, and statements that could result in alternative conceptions [43,44]. According to Chow [15] It appears that many educators and textbook writers are ignorant of the significant cognitive challenges associated with learning algebra. Because of this, many students lack the time to create solid intuitive foundations for algebraic concepts or to relate them to previously acquired pre-algebraic concepts. The result analysis of the pre-test and interview both show that the leading causes of errors and misconceptions are pre-conceived knowledge. According to the literature, the primary reasons for mistakes and misunderstandings are due to superficial knowledge, which was most likely caused by teachers rushing to finish the lengthy curriculum and memorization of rules because of surface comprehension [14].

Students' learning happens in the formal school setting, so entirely depends on how the teacher teaches and the content in the textbook. So, the result analysis of the interview revealed that almost all the students have learned the concepts from teachers and they have used the textbook as the main learning materials. Finally, the leading causes of the misconceptions are teachers and textbooks.

5.3 Advantages of Addressing Misconceptions and Impact of Misconceptions on Students' Achievement

The identification of students' difficulties and misconceptions in algebra is very important which will lead to a better understanding of processes and the quality of learning. Misconceptions are the greatest barrier to learning, hence it is important to address it so that learning can be enhanced, that is why it is important for teachers to identify and eliminate the misconceptions present in their students. McDonald [45] and Das [16] stated that exposing those misconceptions and addressing faulty thinking is the most effective way of dealing by designing effective instructions.

The result from the post-test revealed that there was a significant improvement in the post-test comparing the pre-test item analysis. the mean difference in pre-test and post-test was 7.38 and the two-tailed significance value (p) was 0.000 which was lower than the significant value $p<0.05$, this indicated that there was a statistically significant difference in the post-test. That shows that the test scores in the post-test were significantly higher than the test scores of the pre-test. Aligning with the above result, teachers should acknowledge that learners can overcome misconceptions by planning and consciously providing opportunities for learning through effective teaching strategies [2]. Further, the result revealed that the standard deviation of the means of the pre-test was 5.58 and the standard deviation of the means of the post-test was 7.95. This indicated that with intervention misconceptions can be reduced.

Misconception can be controlled but cannot be eradicated. The timely investigation and intervention could minimize misconceptions. Therefore, the above result revealed that after the intervention the result has significantly improved with a statistical mean difference of 7.38. This study also provided evidence that innovative and timely investigation has a significant effect on addressing and eliminating misconceptions, thus developing a more profound and clearer understanding of various concepts in teaching mathematics.

6. CONCLUSION

Several studies have been undertaken to investigate the prevalence of misconceptions in different subjects and explore different strategies to eliminate the misconceptions in the subject-specific concepts teaching. However, exploring the existence of misconceptions in mathematics, particularly linear equations, has not been undertaken in the context of the Bhutanese Education System. Therefore, the current study was carried out find types of misconceptions faced by students, examine causes of misconceptions impact of misconceptions on students' learning achievement and advantages of addressing the misconceptions for the linear equations.

Algebra is one of the essential components of learning mathematics, which is very much linked to our day-to-day lives. However, the students encounter numerous problems in learning algebra. The misconceptions about solving linear equations are addressed among many algebra-related topics. It is crucial to investigate and address the students' misconceptions on time by providing necessary remediation. One remediation is using the simulation strategy to overcome the misconception of the students.

The mixed method was chosen as the appropriate research design. This research naturally qualified sequential explanatory types of the mixed method as the research strategies. A Mathematics Achievement Test (MAT) was used as a part of the quantitative method, and a semi-structured interview was used as a part of the qualitative method. Using both quantitative and qualitative methods qualifies the overall research method as mixed methods research, with the quantitative method as the dominant one. The data analysis of the pre-test revealed possible misconceptions in the linear equation; further, the semi-structured interview confirmed that there are misconceptions. The post-test data analysis showed that using mobile apps like Photomath, Cymath, and GeoGebra to teach the material helped to correct some of the students' misconceptions. Teaching by using mobile apps has significantly minimized the misconceptions in class ninth students in learning linear equations.

7. RECOMMENDATIONS

This study did not inquire how instruction impacts students' interest or motivation to learn mathematics, specifically algebra. Further research should simultaneously explore the contribution of the instruction to the improvement of affective dimensions in mathematics learning and the construction of mathematics understandings. Such research may provide a complete portrayal of students' conceptual learning with multidimensional evidence from cognitive and affective domains of student learning.

Since all learning involves transfer from prior knowledge and previous experiences, awareness and understanding of a student's initial conceptual framework and topic can be used to formulate more effective teaching strategies. Suppose this idea is taken a step further. In that case, it could be said that since misconceptions comprise part of a conceptual framework, understanding the origins of misconceptions would further facilitate the development of effective teaching strategies. Understanding the source of errors carries significant consequences for how teachers address misconceptions. In order to facilitate more effective student construction of new knowledge, students' misconceptions need to be identified and addressed. Additionally, further research is needed to help teachers understand how students experience conflict, how they feel when they experience cognitive conflict, and how those experiences relate to their final responses because cognitive conflict has both constructive and destructive potential. Thus, by recognizing, interpreting, and managing cognitive conflict, a teacher can successfully interpret his/her students' cognitive conflict and make conceptual change more likely or guide and lead students to have meaningful learning experiences in secondary school algebra.

Finally, based on the study results, the author may suggest the following recommendations:

- Teachers should concentrate in the classroom on the basic concepts and skills associated with the concept of equations and methods of solving.
- Further studies should be conducted in the field of the common error analysis of mathematics students about solving equations of two variables.

CONSENT AND ETHICAL APPROVAL

Prior to starting the study, ethical approval was requested from the ethical College research committee (Samtse College of Education), the Ministry of Education, the Dzongkhag Chief Education Officer (DCEO), the Principal, the willingness of the participants, and a letter of concern from the parents of the participating children. After analysing data, the researchers safely maintained documents both hard and soft in a password-protected three years following the dissertation's final submission.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle5.com/review-history/102185