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Non-Shannon Perspective on Channel Capacity for Binary Source Codes Using Weighted Conditional Entropy

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Discrete memory-free channels with a very low capacity are known as noisy channels. Our recent study has yielded some new insights into the channel capacity of noisy channels, which could prove useful in the development of mathematical models for these channels and other contexts.

Keywords: Non Shannon entropy; channel capacity; weighted entropy; conditional entropy; mutual information; information theory etc.

1 Introduction

By definition, noisy channels are discrete memory-less channels with low capacity. These channels were initially created by Reiffen [1] to explore physical channels operating at low ratios. Researchers [2-4] have

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demonstrated that even in cases where channels are merely functioning at low capacity, these channels nonetheless play a significant role in communication. Majani [5] provided a broad mathematical model of noisy channels and demonstrated that Reiffen's description does not apply to all channels with low capacity. In his classification of noisy channels, he distinguished between two types: type I and type II. All of the Type I Noisy Channels fall within Reiffen's description. Certain Type II Noisy Channels, such as the noisy Z channel, are not covered by Reiffen's definition. Our effort will mostly concentrate on Reiffen's Type I Noisy Channels.

Let as assume that there is a discrete memory less channel with input and output alphabets of X and Y respectively, input probabilities$P(x)$, output probabilities$P(y)$, and transition probabilities $P(y/x)$, and both X and Y are taken to be finite. Reiffen states that the channel is extremely loud if, for all $x \in X$ and $y \in Y$, then we have

$$\frac{P(y)-P(y/x)}{P(y)} = \varepsilon_x(y) \ll 1 \tag{1.1}$$

Equation (1.1) also written as

$$P(y/x) = P(y)(1-\varepsilon_x(y)) \tag{1.2}$$

We observe that

$$\sum_{x\in X} P(x)\,\varepsilon_x(y) = 0 \text{ and } \sum_{y\in Y} P(y)\varepsilon_x(y) = 0 \tag{1.3}$$

with $\varepsilon_x(y) \ll 1$. Later, Gallager [6] proposed that $P(y)$ need only be an approximation of the output probabilities rather than the output probabilities itself.

Information theory has been very helpful for researching information transfer through noisy communication channels [7] because it offers a flexible mathematical framework of Such paths for communication. The concepts of entropy [6], mutual information [6], parametric measures [8-10], multivariate normal distribution [11], divergence [12], directed divergence [13-15], quantitative-qualitative measure [16,17], exponential entropy functional [18 and 19] and many others are derived from information theory. Their generalisations [20, 21, and 22] have been used in the areas of pattern recognition and medical diagnostics, among others. According to Shannon [6], communication over a discrete memoryless channel is always possible as long as the channel capacity is non-zero. The entropy function described by Shannon [23] is given by

$$H_P(X) = -\sum_{x\in X} P(x) lnP(x)$$

The generalised Rènyi entropy can be expressed as follows:

$$H_P(X) = \frac{1}{1-\alpha}\, ln\,(\sum_{x\epsilon X} P^{\alpha}(x)),\;\; \alpha > 0. \tag{1.4}$$

It represents the degree of uncertainty about the input alphabet X. Similar to the output alphabet Y level of uncertainty can be expressed as

$$H_P(Y) = \frac{1}{1-\alpha} ln\,(\sum_{x\epsilon X} P^{\alpha}(y)),\;\; \alpha > 0. \tag{1.5}$$

X and Y are discrete random variables so joint entropy function is given as

$$H_P(X,Y) = \frac{1}{1-\alpha}\, ln\,(\sum_{x\in X}\sum_{y\in Y} P^{\alpha}(x,y)) \tag{1.6}$$

The conditional entropy are defined as

$$H_P(X/Y) = \frac{1}{1-\alpha}\, ln\,(\sum_{x\in X}\sum_{y\in Y} P^{\alpha}(x/y)) \tag{1.7}$$

$$H_P(Y/X) = \frac{1}{1-\alpha}\, ln\,(\sum_{x\in X}\sum_{y\in Y} P^{\alpha}(y/x)) \tag{1.8}$$

The channel capacity C is given by the equation

$$C = \max_{P(x)} I(X, Y) \tag{1.9}$$

where $I(X, Y)$ is mutual information which was given about X by Y or Y by X and is given by

$$I(X, Y) = H_P(Y) - H_P(Y/X) = H_P(X) - H_P(X/Y) = I(Y, X) \tag{1.10}$$

The Shannon entropy function additive property is defined by the fact that the information gain function is logarithmic.

The weighted entropy was proposed by Munteanu and Tarniceriu [24] which form is

$$H_{Pc}(X) = \frac{\gamma}{1-\alpha} \ ln \sum_{x \epsilon X} P^{\alpha}(x) + \delta \sum_{x \epsilon X} P(x)c(x) \tag{1.11}$$

where $c(x)$ denotes the weights assigned to the input alphabet X, with α and β standing in for arbitrary constants that will be decided by boundary conditions. The weighted variants of (5), (6), (7), and (8) are defined similarly as

$$H_{Pc}(Y) = \frac{\gamma}{1-\alpha} \ ln \sum_{y \in Y} P^{\alpha}(y) + \delta \sum_{y \in Y} P(y)c(y) \tag{1.12}$$

$$H_{Pc}(X, Y) = \frac{\gamma}{1-\alpha} \ ln \sum_{x \in X} \sum_{y \in Y} P^{\alpha}(x, y) + \delta \sum_{x \in X} \sum_{y \in Y} P(x, y)c(x, y) \tag{1.13}$$

$$H_{Pc}(X/Y) = \frac{\gamma}{1-\alpha} ln \sum_{x \epsilon X} \sum_{y \epsilon Y} P^{\alpha}(x/y) + \delta \sum_{x \epsilon X} \sum_{y \epsilon Y} P(x, y)c(x \ / \ y) \tag{1.14}$$

$$H_{Pc}(Y/X) = \frac{\gamma}{1-\alpha} ln \sum_{x \epsilon X} \sum_{y \epsilon Y} P^{\alpha}(y/x) + \delta \sum_{x \epsilon X} \sum_{y \epsilon Y} P(x, y)c(y \ / \ x) \tag{1.15}$$

The channel capacity of weighted entropy is given as

$$C = \max_{P(x)} \bar{I}(X, Y) = \max_{P(x)} \big(H_{Pc}(Y) - H_{Pc}(Y/X)\big) = \max_{P(x)} (H_{Pc}(X) - H_{Pc}(X/Y)) \tag{1.16}$$

The physical importance of the quantity $E_0 = -\ln \int \left(\int p(x) \big(p(y|x)\big)^{\frac{1}{2}} dx \right)^2 dy$ is substantial. The exponent in the upper bound of the error probability that corresponds to a zero rate is denoted as E_0. The generalization to discrete input-continuous output channels is simple in the case of discrete channels, though. R_{comp}, the calculation cutoff rate for the sequential decoding process, is defined. The average amount of decoding computations grows algebraically with constraint length rather than exponentially for rates less than R_{comp}. According to Reiffen, $R_{comp} \leq E_0$ for discrete input channels. Verma [8, 14] has made numerous attempts to define the weighted generalizations of the Shannon entropy function presented above; nevertheless, we have taken into consideration the weighted measure provided by (1.12). The channel capacity of class I VNCs was also determined by Reiffen [1] and was expressed as $C = \frac{1}{2}\big(\sum_{x \epsilon X} \sum_{y \epsilon Y} p(x)p(y)\varepsilon_x^2(y)\big)$, $\varepsilon_x(y) \ll 1$components in this equation are correct up to the second order. We have determined the weighted channel capacity of Reiffen's Class I VNCs in the current work. We have also used the Non-Shannon entropy provided by (1.12) to evaluate the channel capacity.

2 Our Results

2.1 Channel capacity of weighted entropy function

The weighted conditional entropy given by (1.15) is

$$H_{Pc}(Y/X) = \frac{\gamma}{1-\alpha} ln \sum_{x \epsilon X} \sum_{y \epsilon Y} P^{\alpha}(y/x) + \delta \sum_{x \epsilon X} \sum_{y \epsilon Y} P(x, y)c(y \ / \ x)$$

If alphabet $x \epsilon X$ is assumed to represent the source codewords and the corresponding weights to be the codeword length, then the constants γ and δ both take values of -1[23] .

We have,

$$H_{Pc}(Y/X) = \frac{-1}{1-\alpha} ln \sum_{x\epsilon X}\sum_{y\epsilon Y} P^{\alpha}(y/x) - \sum_{x\epsilon X}\sum_{y\epsilon Y} P(x,y)c(y\ /\ x)$$
$$H_{Pc}(Y/X) = \frac{-\alpha}{1-\alpha} \sum_{x\epsilon X}\sum_{y\epsilon Y} ln\ P(y/x) - \sum_{x\epsilon X}\sum_{y\epsilon Y} P(x,y)c(y\ /\ x)$$
$$H_{Pc}(Y/X) = \frac{-\alpha}{1-\alpha} \sum_{x\epsilon X}\sum_{y\epsilon Y} ln\ P(y/x) - \sum_{x\epsilon X}\sum_{y\epsilon Y} P(x)P(y/x)c(y\ /\ x)$$
$$= \frac{-\alpha}{1-\alpha} \sum_{x\epsilon X}\sum_{y\epsilon Y} ln\left(P(y)(1-\varepsilon_x(y)\right) - \sum_{x\epsilon X}\sum_{y\epsilon Y} P(x)P(y)(1-\varepsilon_x(y))c(y\ /\ x)$$

Using the formula

$$ln(1-\alpha) = -\alpha - \frac{\alpha^2}{2} - \frac{\alpha^3}{3} \ldots$$

We get

$$H_{Pc}(Y/X) = \frac{-\alpha}{1-\alpha}\left[\sum_{x\epsilon X}\sum_{y\epsilon Y} lnP(y) - \varepsilon_x(y) - \frac{\varepsilon_x^2(y)}{2} - \cdots\cdots\cdots\right] - \sum_{x\in X} P(x)\sum_{y\in Y} P(y)c(y/x) + \sum_{x\in X}\sum_{y\in Y} P(x)P(y)\varepsilon_x(y)c(y/x) \quad (2.1)$$

We have [20], for noisy channels,

$$c(y/x) = c(y) \text{ and } c(x/y) = c(x) \quad (2.2)$$

After some basic changes and using (1.3) and (2.2) in (2.1), we obtain

$$H_{Pc}(Y/X) = \frac{-\alpha}{1-\alpha}\left[\sum_{x\epsilon X}\sum_{y\epsilon Y} lnP(y) - \varepsilon_x(y) - \frac{\varepsilon_x^2(y)}{2} - \cdots\cdots\cdots\right] - \sum_{x\in X}\sum_{y\epsilon Y} P(x)P(y)c(y)$$

The channel capacity of weighted entropy defined as

$$C = \max_{P(x)} \bar{I}\,(X,Y) = \max_{P(x)}\left(H_{pc}(Y) - H_{Pc}(Y/X)\right)$$

$$C = \max_{P(x)}\left[\sum_{y\in Y} P(y)c(y)(\sum_{x\in X} P(x) - 1) + \frac{\alpha}{\alpha-1}\left(\varepsilon_x(y) + \frac{\varepsilon_x^2(y)}{2}\right)\right] \quad (2.3)$$

This is the required channel capacity of weighted entropy function.where the terms in equation (2.3) are true up to the second order of $\varepsilon_x(y) \ll 1$.

3 Conclusion

Here, we evaluated the equations for the channel capacity of Type I Noisy Channels using the weighted conditional entropy for Non-Shannon entropy. By applying Majani's Type II Noisy Channels specification, comparable results might be achieved [12].

Competing Interests

Authors have declared that no competing interests exist.

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