Journal of Advances in Mathematics and Computer Science

Volume 37, Issue 12, Page 84-98, 2022; Article no.JAMCS.94854

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Stochastic Modeling of Inflation and Interest Rates for Defined Benefit Pension Plan Projections in Ghana

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Authors' contributions

This work was carried out in collaboration among all authors. Author ENNN designed the study, authors HDQ, RAL, KDA and ENNN performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors ENNN and HDQ managed the analyses of the study. Authors KDA and ENNN managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2022/v37i121731

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/94854

Received: 18/10/2022

Accepted: 20/12/2022

Published: 26/12/2022

Original Research Article

Abstract

Aims/ Objectives: Pension plan administrators, employers and managers in exchange for service provided currently by employees' pledges stated benefits in the prospective future. For this expense to be budgeted for the future, a pension cost method is used by the plan administrator to establish a form of warranty for the member. The aim of the study was to use deterministic pension plan projection but also considers economic variables that are stochastic, allowing the variables to change in the future randomly to model pension plan projections

Study Design: The study design was cross-sectional.

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Data and Duration of Study: The data used were obtained from the Bank of Ghana as published on their official website. The sample data consists of three hundred and forty-eight (384) observations of monthly inflation rates in Ghana. It covers thirty-one (31) years period spanning from January 1990 to December 2021.
Methodology: Two methods were used to calculate the normal cost, that is, the total and projected unit credit cost using different interest rates and inflation assumptions and constant single life annuity. The economic variables inflation and interest rates were modeled based on data from the Bank of Ghana.
Results: Several time series models were considered, with the seasonal ARIMA (3,1,0)x(2,0,0)12 was the most appropriate time series model for inflation whereas was the best model for interest rate was the nonseasonal ARIMA(1,1,0). Based on the final models selected for the variables, 30 years ahead were forecasted, 100 stochastic simulations were generated on inflation and interest rate variables for the stochastic scenarios. Numerous economic scenarios were generated, 5th, 25th, 50th, 75th and 95th percentiles of probabilities associated with the values were obtained from the cost.
Conclusion: The study revealed that at age 59, the cost under the total unit cost of allocation method had a 0.05 probability of been less than 1.694 and a 0.95 probability that the cost would be lesser than 1.859 and under projected unit cost of allocation method, the cost had a 0.05 probability of been less than 37.284 while 0.95 probability of the cost been less than 45.408 at age 59.

Keywords: Pension; pension cost methods; stochastic modelling; normal cost; actuarial liability.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16.

1 Introduction

Germany was the first country to set up a mandatory pay-as-you-go (PAYG) pension scheme in 1889. A PAYG scheme refers to a plan in which the current pension welfare of the elderly is financed by contribution from the current working population [1]. The key purpose of a pension is to safeguard the fall of an individual's consumption when retiring from active work. Assessing the future and making provision for it, a pension plan is a form of retirement plan, ordinarily tax exempted, where an employer contributes toward a pool of funds to provide for an employee's benefit in the future, is also a way for a prospective retiree to transfer a portion of their current earned income before retirement. Pension plan has regulations and procedures as to how benefits and contributions made are calculated. Pension plan are primarily in two forms: defined benefit and defined contribution plan. For defined benefit (DB), the calculated amount of pension at retirement payable to employees is based on a predetermined formula on the employee's service and salary before retirement, funding the plan is fully the responsibility of employer. Bodie et al, explained defined contribution (DC) planning is an easier retirement plan where steady contribution are paid into the member's retirement account by the employer and occasionally employees. Contributions are generally stated as a prearranged portion of salary, even though that portion fluctuates over the path of member's career. The fund in the retirement account is either be paid out as a lump sum or an annuity to the retiree in the future, the volume of which is dependent on the accrued value. Pension options are made in the future, at retirement and could be affected by wages throughout the career of the employee or heavily affected by the rates of return earned every year during employment, lump sum amount issued are dependent on inflation at retirement, the prevailing interest rate, and/or influenced by other economic variables at retirement. The real amount at retirement depends on the interest rate, the remuneration path of the employee and investment performance of the fund[2]. Economic variables identified based on most global pension plan valuation requirements includes inflation, wage, long-term interest rate, and equity return [3, 4, 5].

The study takes a pension formula to compute the cost of the plan over a member's active work until retirement, the basic principle used: new benefits are earned each year by the active member, this is termed normal cost. Also, this term is explained as a portion of the actuarial present value of future benefits apportioned to a particular year for member or the entire plan. Normal cost under the assumption of inflation and interest rate were considered using the total unit credit and projected credit cost of allocation methods. This research seeks to use the same steps of deterministic pension plan projection but also considers cases where the economic variables are stochastic, allowing the variables to change in the future randomly. In recent decades' rapid growth of pensions plans has attracted widespread attention in business, government, and academic circles, even individuals enrolled under the various plan express much interest related to their pension. It is worthy to note that low interest rates can make it difficult to meet pension obligations. Correspondingly, if markets perform in the coming decades as they have performed in the last century, then most funds are not likely to be in crisis. However, if markets severely underperform, then national pension schemes may need to make significant adjustments to estimate pension liabilities and honouring payments to close the funding gap [6]. The existence and scope of underfunding becomes an actuarial question of what future rate of return states and cities From a pensioner's perspective, the significant question to the pension plan is how thriving will the plan probably perform [7, 4]. The key objective of this study is to make projection of a pension plan for the next 30 years based on key economic variables. The main motivation behind this paper is the need to examine pension plan performance based on economic variables to safeguard the fall of an individual's consumption when retiring from active work. This paper thus contributes to empirical evidence of the research into the behavior of economic series in pensions plan projections in Ghana.

2 Materials and Methods

2.1 Time series and stochastic process

In general, time series models can have several forms and characterize different stochastic processes. A time series model could be linear or non-linear depending on the current value of the series if historical observations have linear or non-linear function.

2.2 Autoregressive moving average model (ARMA)

The combined autoregressive AR (p) and moving average MA (q) model gives an autoregressive moving average ARMA (p, q) model, which is an appropriate model for univariate time series data. The AR (p) model is expressed as:

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + . . . + \varphi_p y_{t-p} + \varepsilon_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t \tag{2.1}$$

where y_t and ε_t are the observed values and random error or shocks at time t, φ_i $(i = 1 , 2 , . . . , p)$ are the parameters of the model, φ_i is the constant term and p is the order of the time series model.

The MA (q) model is given by the equation:

$$y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - . . . - \theta_q \varepsilon_{t-q} = \mu + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2.2}$$

where μ is the mean of the series, θ_i $(i = 1 , 2 , . . . , q)$ represents parameters of the model with order$q$, with random errors ε_t assummed as white noise process.
The mixed autoregressive moving average ARMA (p , q) models are a combination of autoregressive and moving average models which is mathematically presented as:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2.3}$$

where the order (p, q) represents p order for autoregressive AR(p) and q for the moving average MA(q) terms.

ARMA models are regularly employed using lag operators. The rth lag operator is expressed as$L^r y_t = y_{t-r}$; $r = 1 , 2 , . . .$, and the lag polynomial representation of the autoregressive AR(p) , moving average

MA(q) and ARMA(p , q) models are respectively stated as:

AR (p) model : $\varphi(L)y_t = \varphi_0 + \varepsilon_t$;

MA (q) model: $y_t = \mu + \theta(L)\varepsilon_t$;

ARMA (p, q) model: $\varphi(L)y_t = \mu + \theta(L)\varepsilon_t$;

where $\varphi(L) = 1 - \sum_{i=1}^{p} \varphi_i L^i$ and $\theta(L) = 1 - \sum_{i=1}^{p} \theta_i L^i$ are the respective characteristic polynomials of the AR(p) and MA(q) of orders p and q.

It can be shown that (Box, Jenkins & Gwilym, 1970; Hipel & McLeod, 1994; Rao, 2022), a significant property of AR (p) technique is the invertibility condition, that is, an autoregressive AR (p) process can be written as an infinite moving average MA (∞) process.

2.3 Autoregressive integrated moving average model (ARIMA)

An Autoregressive Integrated Moving Average (ARIMA) model forecasts the outcome values as a linear combination of the time series' own previous values, historical shocks, present and historical values of other time series [8, 9, 10, 11].

The ARIMA(p, d, q)model using lag polynomials mathematically can be written as:

$$\varphi(L)(1-L)^d y_t = \mu + \theta(L)\varepsilon_t$$

$$(1 - \sum_{i=1}^{p} \varphi_i L^i)(1-L)^d y_t = \mu + (1 - \sum_{i=1}^{p} \theta_i L^i)\varepsilon_t \quad (2.4)$$

where p , d and q are the order of autoregressive, integrated, and moving average terms of the model respectively which could be any number not less than one, d is the differencing required to achieve stationarity [8, 9, 10, 11].

2.4 Seasonal autoregressive integrated moving average (SARIMA) models

Box and Jenkins (1970) in dealing with seasonality in a process proposed a generalized model called the Seasonal Autoregressive Integrated Moving Average (SARIMA) model, the seasonal differencing order of the model is used to eliminate non-stationarity from the series. A first order seasonal difference is expressed mathematically as $z_t = y_t - y_{t-s}$. For $s = 12$ and $s = 4$ for monthly and quarterly time series respectively. The model is generally presented as $SARIMA(p, d, q) \times (P, D, Q)_s$ and in lag polynomial operator form as:

$$\Phi_P(L^s)\varphi_p(L)(1-L^s)^D(1-L)^d y_t = \Theta_Q(L^s)\theta_q(L)\varepsilon_t \quad (2.5)$$

$$\Phi_P(L^s)\varphi_p(L)z_t = \Theta_Q(L^s)\theta_q(L)\varepsilon_t \quad (2.6)$$

Where z_t is the regular as well as seasonally differenced series of y_t .

2.5 Estimation of model parameters

The maximum likelihood technique of estimation is employed to approximate the model's parameters. A brief description of how this method works is outlined as follows.

We let x_1 , x_2 , ... , x_n be n observed values of the random variable X, presumed to follow a probability density function $f_X(x)$, whose m parameters are θ_1 , θ_2 , ... , θ_m. Using maximum likelihood estimation procedure, the estimated values for the parameters are those values that maximize the likelihood function $L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m)$ which is defined by:

$$L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m) = \prod_{i=1}^{n} f_{X_i}(x_i) \tag{2.7}$$

The natural logarithm of the likelihood function is a monotonically increasing transformation of the likelihood function, and also a product of positive values less than 1 which gets inconveniently small, instead of maximizing the likelihood function itself, it is more convenient to maximize the log-likelihood function,

$$Ln\ L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m) = Ln\ \prod_{i=1}^{n} f_{X_i}(x_i) = \sum_{i=1}^{n} Ln\ f_{X_i}(x_i) \tag{2.8}$$

The maximum likelihood estimator (MLE) of each of the model parameters is obtained by differentiating $Ln\ L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m)$ with respect to each of the parameters and setting to zero, yielding m equations in m unknown parameters.
Solving the first order conditions:

$$\frac{\partial}{\partial\theta_i} Ln\ L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m) = 0\ \ ,\ \ i = \ 1\ ,\ 2\ ,\ \ .\ .\ .\ .\ ,\ m,$$

yields, potentially nonlinear equations in $m$unknown values $\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m$:

$$\frac{\partial}{\partial\boldsymbol{\theta}} Ln\ L(\boldsymbol{\theta}) = \begin{pmatrix} \frac{\partial}{\partial\theta_1} Ln\ L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m) \\ \vdots \\ \frac{\partial}{\partial\theta_m} Ln\ L(\theta_1\ ,\ \theta_2\ ,\ \ .\ .\ .\ .\ ,\ \theta_m) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.9}$$

2.6 Benefit cost allocation methods for projection

Cost methods/techniques for actuarial practice are generally classified in two main forms: Cost and Benefit methods of allocation. Benefit method of allocation apportions various planned years to the benefit due to be received from the plan and there after actuarial present value of the benefit at the time are determined or computed while that of the cost method of allocation assigns actuarial present value to all projected/anticipated future benefit to definite time periods without apportioning benefit. Total and projected unit credit methods of cost allocations are classified as benefit allocation cost methods [4, 5].

2.7 Total unit credit cost of allocation method

Total unit credit cost of allocation method for an accumulated/accrued benefit is regularly used when the plan defines a constant present annual percentage wage for the member or a flat Cedi amount of accrued annual benefit. Benefit apportioned to a year is expressed as the projected/estimated growth in the member's accrued plan benefit in that year. An annuity due beginning at a said age k, with a discounted interest rate and assumed survival to age multiplied by accumulated or accrued benefit termed Actuarial liability is mathematically formulated as:

$$^{k}(Al)_x = \ B_x\ {}_{k-x}P_x\,(T) d^{k-x} \ddot{a}_k \tag{2.10}$$

The normal cost under the total unit credit cost of allocation method is explained as an annuity due, beginning at a said age k with a discounted interest rate and assumed survival to age$x$multiplied by the growth in a member's benefit during a year in the plan and mathematically present as:

$$^{k}(Nc)_x = \ b_x\ {}_{k-x}P_x\,(T) v^{k-x} \ddot{a}_k \tag{2.11}$$

Assumptions about the member's attained age wage was made as salary grows each year due to productivity of the company and inflation that is a percentage increase in productivity and a percentage inflation growth. We assume an entry age of thirty years with a wage of one Cedi, then the wage/salary at some age thereafter is computed with the formula:

$$s_x = [(1+1)(1+p)]^{x-y} \tag{2.12}$$

where s_x is the wage at age x, x and y are current and entry age of the participant respectively, I rate of inflation rate and $p$is the productivity growth rate.

2.8 Projected unit credit cost of allocation method

Here, same actuarial assumptions are applied with differences between methods being benefit apportionment to each year[4, 5]. The accumulated benefit at the start of age x is ascertained by the benefit projected at time of retirement, this benefit at the time is pro-rated over the number of service years. Accumulated benefit is then expressed mathematically as:

$$B_x = \frac{B_k}{(k-y)}(x-y) \tag{2.13}$$

Where B_k the projected accumulated benefit at time of retirement, k is the predetermined age by the plan for retirement and y is the entry age. The actuarial liability for projected unit credit method is:

$$^{k}(AL)_x = \frac{x-y}{k-y}\, B_x\ {}_{k-x}P_x\,(T) v^{k-x} \ddot{a}_k \tag{2.14}$$

And the actuarial liability is a portion of the present value of the future benefits with present value of future benefits given as [4, 5] :

$$^{k}(PVFB)_x = B_x\ {}_{k-x}P_x\,(T) v^{k-x} \ddot{a}_k \tag{2.15}$$

The normal cost for the projected unit cost method is mathematically given as:

$$^{k}(NC)_x = \frac{B_x}{k-y}\ {}_{k-x}P_x\,(T) v^{k-x} \ddot{a}_k \tag{2.16}$$

3 Data, Results and Discussion

The data used were obtained from the Bank of Ghana as published on their official website. The sample data consists of three hundred and forty-eight (348) observations of monthly inflation rates in Ghana. It covers thirty-one (31) years period spanning from January 1990 to December 2021.

The results shown for the data of the monthly inflation rate obtained a mean of 21.64 with standard error of 0.8 and a standard deviation of 13.49. Different models were fitted, with the best carefully chosen based on the minimum values of the AIC, BIC and the Log Likelihood maximum value and the corresponding significance tests. $SARIMA\ (3,1,0) \times (2,0,0)_{12}$ was the most appropriate model based on the selection criterion with AIC of 1,096.78, BIC of 1,118.73 were the minimum among all the fitted models and a maximum log likelihood of -542.39 and these satisfy all of the selection criteria [12].

Model parameters were then estimated with all the parameters being significant at $(1-\alpha)\,100\%$ confidence level. Treasury bills rate is used in the study as interest rate since in Ghana most of the Bank and Non-bank financial institutions depend solely on this rate for the interest rate on their investment packages. The 182-day Treasury bills rate from Jan 2006 to Dec 2021 were used as they are the intermediate between the 91-day and 365-day bills. Data obtained from the Bank of Ghana (BoG) consisted of 194 observations of the monthly 182-day T-bill rate. Preliminary analysis of the data yielded a mean of 17.57, standard error of 0.65 and standard deviation of 6.74. $ARIMA\ (1,1,0)$ was the suitable model centered on the selection criteria among the different ARIMA models fitted for the data series since it has the smallest BIC value. Even though the AIC value of the $ARIMA\ (1,1,0)$ model was larger than other models fitted, the BIC criterion was used for selection because the BIC criterion is a consistent estimator and tends to select models with fewer parameters satisfying the principle of parsimony as compared to AIC criterion. Model parameters were then estimated with all the parameters being significant at $(1-\alpha)\,100\%$ confidence level.

3.1 Pension plan projections methodology

Defined Benefit pension plan projection is essentially a series of valuations at each valuation year which normally ranges from one year to three years into the future. In this research, the deterministic approach was first applied where the rates for the economic variables were chosen and the stochastic process was applied to allow for changes in the future. Here, a pension plan with only one member having constant demographic profile is studied; we assume an entry age of 30 years and the member retires at age 60. Post-retirement period was ignored; we also employed the projected unit credit cost of allocation of funds method and the total unit credit cost of allocation to determine the normal cost of the plan. The study performs growing-concern valuations only and assumes that the accumulation of the benefit is from contribution to the fund.

Deterministic cases were performed for inflation and interest rate as economic variables under varying assumptions for both the projected unit credit cost of allocation and the total unit credit cost of allocation methods and later consideration was given to the case where the variables were assumed random after obtaining stimulated data using the final models selected for inflation and interest rate.

3.2 Total unit credit cost of allocation method

The computed salaries of attained age with a linking graph from entry age of 30 years till retirement under a total unit credit cost of allocation are shown in Table 1.

Table 1. Salaries attained age as a percentage of Entrant Age of 30 years

Age	Salary	Age	Salary	Age	Salary
30	1.0000	41	1.5443	52	2.3850
31	1.0403	42	1.6066	53	2.4811
32	1.0822	43	1.6713	54	2.5811
33	1.1258	44	1.7387	55	2.6851
34	1.1712	45	1.8088	56	2.7933
35	1.2184	46	1.8816	57	2.9059
36	1.2675	47	1.9575	58	3.0230
37	1.3186	48	2.0364	59	3.1448
38	1.3717	49	2.1184		
39	1.4270	50	2.2038		
40	1.4845	51	2.2926		

A 1.5 proportion of the final five-year average salary was used to compute benefit accrued, the standard to an annual pension benefit at normal retirement of final average earnings times the employee's years of service. Under the total unit credit cost of allocation method, benefit accrual $b_x$is computed from entry to retirement of each age. To compute b_x, accrued benefit is firstly computed $B_x$where

$$B_x = 0.015(x-y)\left(\frac{1}{x-y}\right)\sum_{t\,=\,x-y}^{k-1} s_t$$

k represents the age at retirement and $s_t$represent the attained age salary of a member age t, $b_x$is theB_{x+1} minusB_x.

The pension plan assumes a single age retirement at sixty and to calculate the accumulated benefit $B_x$for the participant with age entry thirty years that has attained thirty one and thirty two years as $B_{31} = 0.015(31 - 30)(2.1)\{1\} = 0.015$,

$$B_{32} = 0.015(32-30)(0.5)\{1+1.04\} = 0.0306$$

respectively and the benefit accrual at age thirty-one as

$$b_{31} = B_{32} - B_{31} = 0.0306 - 0.015 = 0.0156.$$

For an entrant aged thirty, the accrued benefit and the benefit accrual under the total unit credit cost of allocation method is shown in the Table 2.

Table 2. Total Unit Credit Cost of Allocation Accrued Benefit and Benefit Accrual Functions as a percentage of Entrant Age of 30

Age	Benefit Accrual b_x	Accrued Benefit B_x	Age	Benefit Accrual b_x	Accrued Benefit B_x	Age	Benefit Accrual b_x	Accrued Benefit B_x
30	0.0150	0.0000	41	0.0375	0.2504	52	0.0799	0.8579
31	0.0156	0.0150	42	0.0406	0.2880	53	0.0848	0.9377
32	0.0162	0.0306	43	0.0439	0.3286	54	0.0897	1.0226
33	0.0171	0.0468	44	0.0472	0.3725	55	0.0950	1.1122
34	0.0173	0.0639	45	0.0508	0.4197	56	0.0944	1.2072
35	0.0223	0.0812	46	0.0544	0.4706	57	0.0997	1.3017
36	0.0244	0.1035	47	0.0583	0.5250	58	0.1052	1.4013
37	0.0267	0.1279	48	0.0622	0.5833	59	0.1109	1.5065
38	0.0292	0.1546	49	0.0664	0.6455	60		1.6174
39	0.0319	0.1838	50	0.0707	0.7119			
40	0.0347	0.2157	51	0.0752	0.7827			

The interest rate assumptions are defined as three percent inflation, two percent risk premium and a unit percent risk free rate, in order to calculate the discounted rate and this aggregates to a six percent interest rate. The discounting function is computed as $v_t = \left(\frac{i}{1+i}\right)^t$.

Table 3. Discount function values for t Years

Age	Discount	Age	Discount	Age	Discount
30	0.1741	40	0.3118	50	0.5584
31	0.1846	41	0.3305	51	0.5919
32	0.1956	42	0.3503	52	0.6274
33	0.2074	43	0.3714	53	0.6651
34	0.2198	44	0.3936	54	0.7050
35	0.2330	45	0.4173	55	0.7473
36	0.2470	46	0.4423	56	0.7921
37	0.2618	47	0.4688	57	0.8396
38	0.2775	48	0.4970	58	0.8900
39	0.2942	49	0.5268	59	0.9434

Starting at age sixty the present value of a life annuity due is the last element computed with assumption of 6% continued interest rate, the annuity value is 11.145%. The normal cost of future retirement benefits at age thirty of the normal cost is calculate as

$$\begin{aligned} {}^{60}\left(NC\right)_{30} &= b_{30}{}_{30}P_{30}^{(T)} v^{30} \ddot{a}_{60} \\ &= (0.015 \times 0.0238 \times 0.174 \times 11.145) \\ &= 0.00069 \end{aligned}$$

Table 4. Total Unit Credit Cost of Allocating Normal Costs as a Percentage of Attained Age Salary of an Entrant Age of 30

Age	Cost	Age	Cost	Age	Cost
30	0.069	41	0.329	52	1.329
31	0.076	42	0.378	53	1.496
32	0.084	43	0.433	54	1.677
33	0.094	44	0.493	55	1.883
34	0.101	45	0.563	56	1.984
35	0.138	46	0.639	57	2.219
36	0.159	47	0.725	58	2.483
37	0.186	48	0.820	59	2.775
38	0.215	49	0.928		
39	0.249	50	1.048		
40	0.287	51	1.181		

The total unit credit cost of allocation method of the normal cost for an age thirty as a percent of the attained age salary is shown with a linking graph.

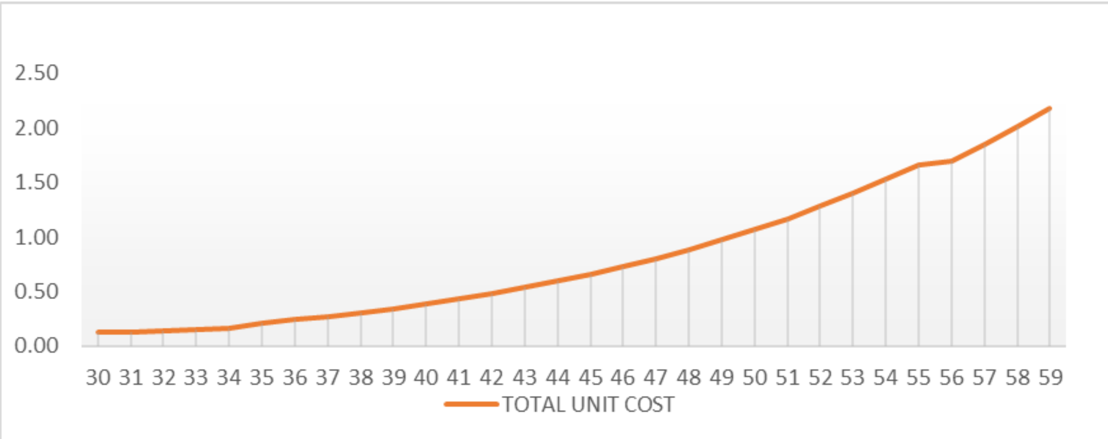

Fig. 1. Total Unit Credit Cost of Allocation Normal Costs as a Percentage of Attained Age Salary

Costs as a percentage of attained salary rises rapidly in the latter years, since salary progressively increases due to inflation, experience and productivity among other factors, and the normal costs is even more with a rise in the Cedi amount.

3.3 Projection Unit Credit Cost of Allocation Method

With the assumption of the model plan, the accrued benefit estimated at retirement was 2.78 as observed in the calculations in Table 4 of the total unit credit cost of allocation method. The projected unit credit cost of allocation method as a percentage of attained age salary of an entrant aged 30 with relation to the normal cost is shown in Table 5 with its accompanying graph in Fig. 2.

The projected unit credit cost of allocation of the normal cost does not sharply increase compared to the total unit credit cost of allocation method.

Normal cost for the projected unit cost method at age thirty and thirty-one were computed as

$$^{60}\left(NC\right)_{30} = \frac{B_{60}}{60-30}{}_{60-30}p_{30}^{(T)}v^{60-30}\ddot{a}_{60} = \quad 0.025$$

Table 5. Projected Unit Credit Cost of Allocation Normal Costs as a Percentage of Attained Age Salary of an Entrant Age of 30

Age	Cost	Age	Cost	Age	Cost
30	0.25	41	0.75	52	3.37
31	0.27	42	0.84	53	4.08
32	0.30	43	0.94	54	5.05
33	0.33	44	1.06	55	6.42
34	0.36	45	1.20	56	8.51
35	0.40	46	1.36	57	12.03
36	0.44	47	1.55	58	19.12
37	0.49	48	1.78	59	40.54
38	0.54	49	2.06		
39	0.60	50	2.40		
40	0.67	51	2.83		

3.4 Stochastic modelling of defined Benefits

To find the probability of the normal cost at retirement, several economic scenarios were generated from the stimulation based on the models selected for the economic variables forecasted 30 years ahead, then a 100-random sampling for stochastic scenarios on inflation and interest rate variables were applied, the difference in the annual averages were used as there could be either a decrease or increase in inflation and interest rate under study. The same steps as that above were performed with the exception that economic variables were now stochastic, meaning the economic series are allowed to change in the future randomly.

3.5 Total Unit Credit Cost of Allocation Method under Stochastic Modelling

With the total unit credit cost of allocation method, assumption sets were the same, member's wage may increase, that is, grows each year due to rate of inflation from the set of data and a one percent company productivity increase. The attained age salaries for an age thirty entrant until retirement is calculate, 1.5% for a final five-year wage average is assumed for the modelling of the pension plan benefit. The pension plan assumes a single age retirement at sixty, interest rate assumptions are dependent on the difference in the annual average from the forecasts and used to calculate the discount rate and present value of a life annuity due is the last element computed with assumption of 6% continued interest rate, the annuity value is 11.145%. The normal costs scale for each age before retirement under total unit credit cost of allocating normal cost as a percentage of attained age salary for the 100 randomly simulated data is given in Fig. 3.

The graph shows normal cost for each age under the varying assumptions for inflation and interest rate randomly simulated from the forecast data by the final time series models. The result from the graph show the fluctuating movement of the normal cost at each age, as shown, at age thirty normal cost is likely to fall within a range of 0.33% to 1.32% whereas at age fifty-nine normal cost will likely range between 1.69% and 1.98%.

Normal costs scale for an entry age of thirty under the total unit credit cost of allocation method as a percentage of the attained age salary from the forecast data randomly simulated is given in Fig 3.
Result from the graph show the movement of the normal of the 100 generated scenarios of the 30-year period normal cost under total unit credit cost of allocation method, as shown, first scenario normal cost starts at

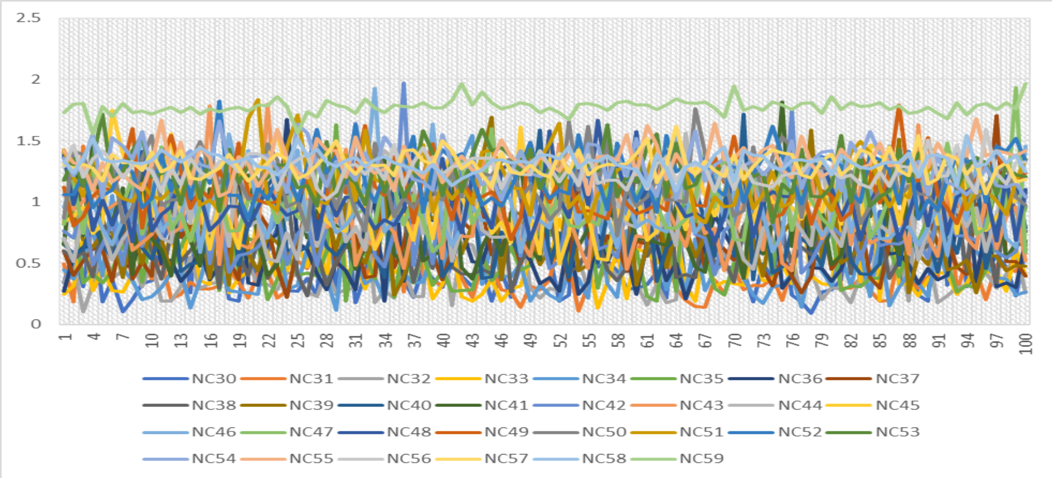

Fig. 2. Normal Cost as a Percentage of each Attained Age Salary for 100 simulations under Total unit cost method

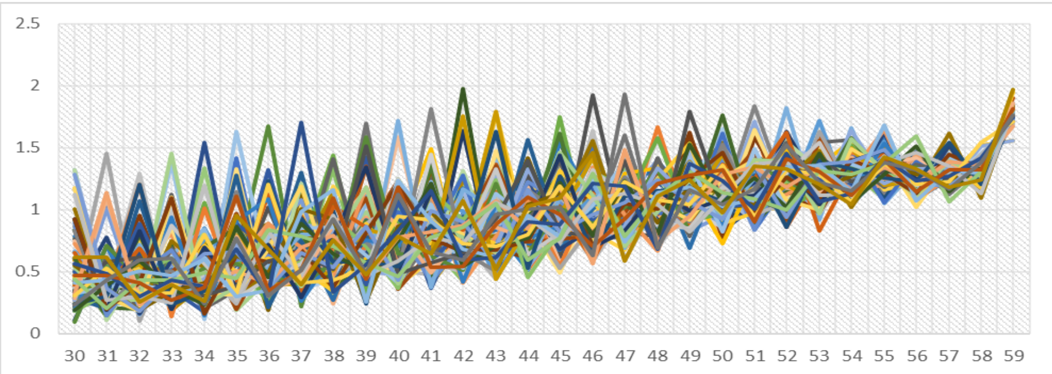

Fig. 3. Total Unit Credit Cost of Allocation Normal Cost as a percentage of attained age salary for 100 simulations

0.33% at age thirty and ends at 1.72% for age fifty-nine, the 62^{th} scenario also shows a start at 0.42%, ends at 1.75% and the 100^{th} scenario resulted in 0.61% as the start with 1.96% as end for ages thirty and fifty-nine respectively.

3.6 Projection Unit Credit Cost of Allocation Method under Stochastic Modelling

The normal costs scale for each age before retirement under projected unit credit cost of allocation method as percentage of attained age salary for the 100 randomly simulated data is given in Fig 4. The graph shows normal cost for each age under the varying assumptions for inflation and interest rate randomly simulated from the forecast data by the final time series models. The results from the graph show the fluctuating movement of the normal cost at each age, as shown, at age thirty normal cost is like to fall within a range of 0.42% to 4.92% whereas at age fifty-nine it is likely to range between 34.58% and 48.74%.

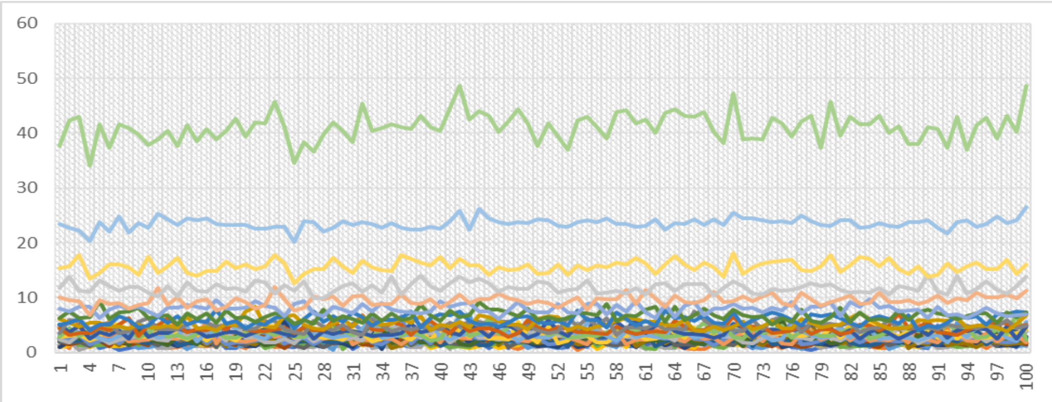

Fig. 4. Normal Cost as a percentage of each attained age salary for 100 simulations under projected unit cost method

The normal costs scale for an entry at age thirty under the projected unit credit cost of allocations method as percentage of attained age salary from the forecast data randomly simulated is given in Fig 5.

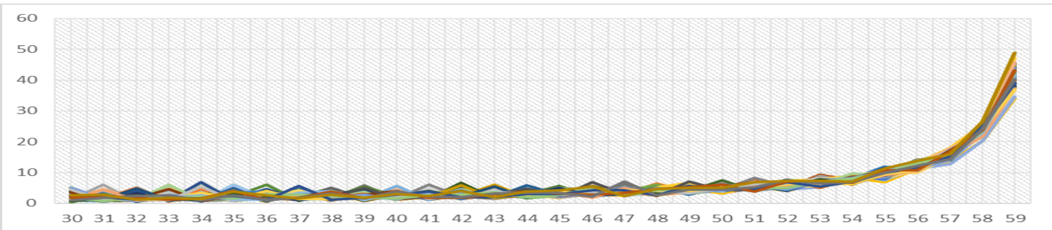

Fig. 5. Projected unit credit cost of allocation normal cost as percentage of attained age salary for 100 simulations

The result from the graph show the movement of the normal cost of the 100 generated scenarios of the 30-year period under the projected unit credit cost of allocation method, as shown, first normal scenario cost starts at 1.29% at age thirty and ends at 43.07% for age fifty-nine, the 62^{nd} scenario also shows a start at 1.65%, ends at 40.41% and the 100^{th} scenario resulted in 2.68% as the start and 48.74% as the end for ages thirty and fifty-nine respectively.

The values involving the future were obtained, for each year, the cost value could either go up or down, these were the scenarios generated, probability distributions were then modelled on these scenarios. The mean for each age attained were obtained as predictions but the information was insufficient, the 5^{th}, 25^{th}, 50^{th}, 75^{th} and 95^{th} percentiles were computed as probabilities associated with the values obtained of the cost. The probabilities are given in Tables 6 and 7 for the total and projected unit credit cost of allocation methods.

From Table 6, at age 30, the 5% percentile was 0.186, this implies that the probability that the cost would be lesser that 0.186 is 0.05, this explains the reason why tomorrow's cost could drop to 0.1 but the probability is 0.05 and the 95% percentile, the probability that cost would be greater than 1.175 is less than 5% or the probability that the cost will be less than the value 1.175 is 0.95.

With the ending cost for the plan at retirement, at age 59, the cost under the total unit cost method would have a 0.05 probability of been less than 1.694 and a 0.95 probability that the cost would be lesser than 1.859 implying that we have a 0.05 probability of the cost been higher than 1.859.

Table 6. Percentiles of total unit credit cost of allocation of normal costs for an entry age of 30 as a percentage of attained age salary

Age	5%	25%	50%	75%	95%
30	0.186	0.281	0.408	0.612	1.175
31	0.188	0.289	0.377	0.518	1.008
32	0.180	0.281	0.395	0.539	0.894
33	0.206	0.319	0.422	0.611	1.002
34	0.187	0.285	0.387	0.572	0.857
35	0.243	0.361	0.569	0.739	1.110
36	0.271	0.395	0.539	0.717	1.091
37	0.355	0.489	0.645	0.814	1.120
38	0.343	0.456	0.598	0.802	1.142
39	0.364	0.500	0.692	0.881	1.342
40	0.389	0.529	0.710	0.950	1.235
41	0.424	0.618	0.777	0.986	1.350
42	0.481	0.606	0.746	0.941	1.242
43	0.479	0.672	0.831	1.056	1.419
44	0.527	0.742	0.842	1.037	1.343
45	0.611	0.817	0.955	1.188	1.461
46	0.629	0.786	0.883	1.121	1.501
47	0.695	0.824	1.045	1.197	1.418
48	0.691	0.877	1.008	1.152	1.402
49	0.829	0.984	1.102	1.240	1.528
50	0.854	1.010	1.133	1.317	1.512
51	0.916	1.015	1.134	1.346	1.497
52	0.937	1.054	1.206	1.358	1.546
53	0.978	1.126	1.251	1.363	1.547
54	1.056	1.177	1.298	1.388	1.565
55	1.097	1.248	1.320	1.404	1.551
56	1.094	1.179	1.273	1.328	1.443
57	1.169	1.229	1.303	1.387	1.452
58	1.192	1.285	1.328	1.373	1.443
59	1.694	1.746	1.778	1.804	1.859

Table 7. Percentiles of projected unit credit cost of allocation normal costs for an entry age of 30 as a percentage of attained age salary

Age	5%	25%	50%	75%	95%
30	0.736	1.136	1.657	2.460	4.539
31	0.783	1.147	1.559	2.074	4.072
32	0.750	1.174	1.638	2.255	3.647
33	0.867	1.355	1.775	2.493	4.075
34	0.836	1.271	1.691	2.511	3.965
35	0.844	1.292	2.078	2.657	4.100
36	0.938	1.358	1.940	2.562	3.931
37	1.175	1.725	2.196	2.805	3.831
38	1.186	1.544	2.013	2.747	3.852
39	1.221	1.681	2.318	2.894	4.596
40	1.325	1.746	2.388	3.209	4.070
41	1.440	2.074	2.619	3.193	4.396
42	1.521	2.014	2.482	3.076	4.306
43	1.555	2.226	2.751	3.474	4.787
44	1.781	2.492	2.853	3.477	4.531
45	2.090	2.732	3.199	3.955	5.009
46	2.204	2.760	3.144	3.986	5.425
47	2.580	3.004	3.716	4.269	5.117
48	2.572	3.236	3.801	4.444	5.291
49	3.280	3.807	4.287	4.890	5.947
50	3.590	4.202	4.791	5.491	6.214
51	4.085	4.594	5.082	5.869	6.754
52	4.510	5.217	5.915	6.684	7.466
53	5.422	6.035	6.611	7.263	8.274
54	6.493	7.285	8.025	8.469	9.346
55	8.063	8.898	9.470	10.041	11.175
56	10.504	11.076	11.731	12.509	13.795
57	14.155	14.939	15.678	16.352	17.617
58	22.142	22.971	23.570	24.100	25.040
59	37.284	39.318	41.186	42.872	45.408

The cost under the projected unit credit cost of allocation method had a 0.05 probability of been less than 37.284 while we obtained a 0.95 probability of the cost been less than 45.408 at age 59. Hence, the ending cost is likely not going to be more than 45.408 since it comes with a probability of 0.05.

4 Conclusions

The paper focused on inflation and interest rates for pension plan projections. Time series models were used to model the economic variables, the final model for inflation was a Seasonal Autoregressive Integrated Moving Average model $SARIMA(3,1,0) \times (2,0,0)_{12}$ and the Autoregressive Integrated Moving Average model $ARIMA$ $(1,1,0)$ for the interest rate. The paper first applied deterministic approach and later stochastic approach to the economic variables to a pension plan with only one member having constant demographic profile; assuming an entry age of 30 and retirement age of 60. Post-retirement period was ignored; the researcher performed a growing-concern for valuation only. The probability of the normal cost at retirement were found looking at the economic scenarios generated with simulation.

The results were dependably on the models selected based on the period of past experience of the economic variables. Numerous scenarios were generated for the parts involving the future, for each year the cost could either go up or down, the cost mean(s) for each age attained were obtained for the method as predictions but the information given by the mean(s) were insufficient, the 5^{th}, 25^{th}, 50^{th}, 75^{th} and 95^{th} percentiles of probabilities associated with the values were obtained of the cost. For the plan projection, the total and projected unit credit cost technique were used under various assumption as way of comparison. (Grizzle, 2005), stated that both techniques have their own flaws, however said, the normal costs given by the projected unit credit is satisfactory. The projection highlights an ending normal cost for the defined benefit plan under the total unit cost method had a 0.05 probability of been less than 1.694 and a 0.95 probability that the cost would be lesser than 1.859 and for the projected unit cost technique, cost had a 0.05 probability of been less than 37.284 while 0.95 probability of the cost been less than 45.408 at age retirement.

Disclaimer

Some part of this manuscript was previously presented and published in the conference: CBAS Annual Science Development Platform Theme: Science for Development: Ghana Asks, Legon Answers on 25-27 April, 2018 At: University of Ghana, Legon. Web Link of the proceeding:
https://www.researchgate.net/publication/344927290_Dela_Hanson_et_al_manuscript_2017.

Acknowledgement

This Research work has been supported by funds from the Carnegie Banga-Africa Project, University of Ghana.

Competing Interests

Authors have declared that no competing interests exist.

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