



## **Simulation Model for Assessment of Non Deterministic Inventory Control Techniques**

**Gladys Bonilla-Enriquez<sup>1</sup> and Santiago-Omar Caballero-Morales<sup>2\*</sup>**

<sup>1</sup>*Instituto Tecnológico de Puebla, Del Tecnológico 420, Corredor Industrial la Cienega, Puebla, Pue., Mexico.*

<sup>2</sup>*Universidad Popular Autónoma del Estado de Puebla A.C. 17 Sur 901, Barrio de Santiago, 72410, Puebla, Pue., Mexico.*

### **Authors' contributions**

*This work was carried out in collaboration between both authors. Author GBE designed the study, researched and analyzed the technical background and wrote the first draft of the manuscript. Author SOCM developed the programming code and revised the technical background and the final version of the manuscript. Both authors read and approved the final manuscript.*

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### **ABSTRACT**

Inventory procurement is one of the most important aspects of the supply chain. This aspect is supported by specific numerical techniques to determine the optimal lot sizes to ensure prompt supply without increasing operational costs for retailers and providers. However, the practical performance of these techniques is only evaluated after implementation, which can lead to unnecessary risks and costs. In such case, simulation can provide the means to evaluate the dynamic performance of these techniques to reduce this risk. The present work describes the development of an open computational simulation code for two non-deterministic inventory control techniques. The functionality of this code provides insights regarding the disadvantages of considering just the static numerical parameters of the model and the advantages of the dynamic aspect of simulation. This contribution can be used by the academic researcher or professional within the logistic field to evaluate and improve their strategies for inventory procurement.

\*Corresponding author: E-mail: [santiagoomar.caballero@upaep.mx](mailto:santiagoomar.caballero@upaep.mx);

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## 1. INTRODUCTION

Inventory management is as important aspect of distribution as proper inventory levels are required to ensure the constant supply of goods. This, however, must comply with specific restrictions to avoid unnecessary risks and costs associated to the inventory supply processes. The concept of an "economic order quantity" (EOQ) has been widely used to define the optimal lot size required to minimize operational costs associated to ordering and keeping goods thorough the supply chain. Defining this lot size is a complex task due to the different variables involved in the inventory supply processes such as costs, delivery times, planning horizon, cycle time, stock-out costs and probabilities, service levels, demand patterns [1].

Thus, different mathematical models have been developed to determine the EOQ to ensure prompt supply without increasing operational costs for retailers and providers and considering the associated variables [1-5].

Two of the most widely used models are the Periodic ( $P$ ) [6] and Continuous ( $Q, R$ ) [7,8] review models with uncertain (variable) demand. While research has been performed on the extension and development of these and new models, this is frequently restricted to the mathematical formulation and numerical evaluation through probabilities and fixed data.

The efficiency of the model is frequently evaluated by a sensitivity analysis, which is focused on determining how the uncertainty in the output of the inventory model can be attributed to the different sources of uncertainty of its inputs [9,10]. However, this requires output evaluations considering different numerical values for the input variables. Still, assessment is performed on static patterns.

Hence, the present work describes the development of an open source computational code to evaluate the efficiency of these non-deterministic inventory control techniques by dynamic simulation. By enabling the user to change the input parameters of the model, this simulation code can determine its suitability under variable demand patterns, thus, providing insights regarding the disadvantages of

considering just the static numerical parameters of the model, and the advantages of the dynamic aspect of simulation. This contribution can be used by the academic researcher, professional or small enterprise within the logistic field to evaluate and improve their strategies for inventory procurement.

## 2. LITERATURE REVIEW AND TECHNICAL BACKGROUND

### 2.1 Simulation as Optimization Tool for Dynamic Processes

The dynamic assessment of the model is frequently performed with the real implementation. However, this may cause unnecessary additional costs and waste of resources, and the use of technological tools represents an advantage to visualize future scenarios and functionality. In some cases, computer simulation has been used to visualize and evaluate inventory replenishment mechanisms. In [11] simulation with the software *AweSim* was considered to obtain more useful and practical results for a continuous review strategy. In [12] a simulation model in the software *Arena* was built to evaluate the impact of maritime transport on the inventory levels of an oil supply chain. In [13] a simulation model was built with the software *Enterprise Dynamics* to develop a continuous-review, base-stock inventory model with general compound demands, random lead times, and lost sales. In [14] simulation-based analysis of inventory strategies in lean supply chains was performed.

While the detailed steps for these procedures are presented in the specialized literature such as in [15], the source data such as the software models, programming code, and data sets, are not explicitly available. Also, license restrictions regarding the implementation software avoids the use of the simulation models for commercial purposes. The high costs of some of these licenses restrict their use by micro, small and medium enterprises which have limited economic resources.

This is the reason to provide a detailed implementation code to compute the parameters of an inventory supply strategy model and evaluate its performance.

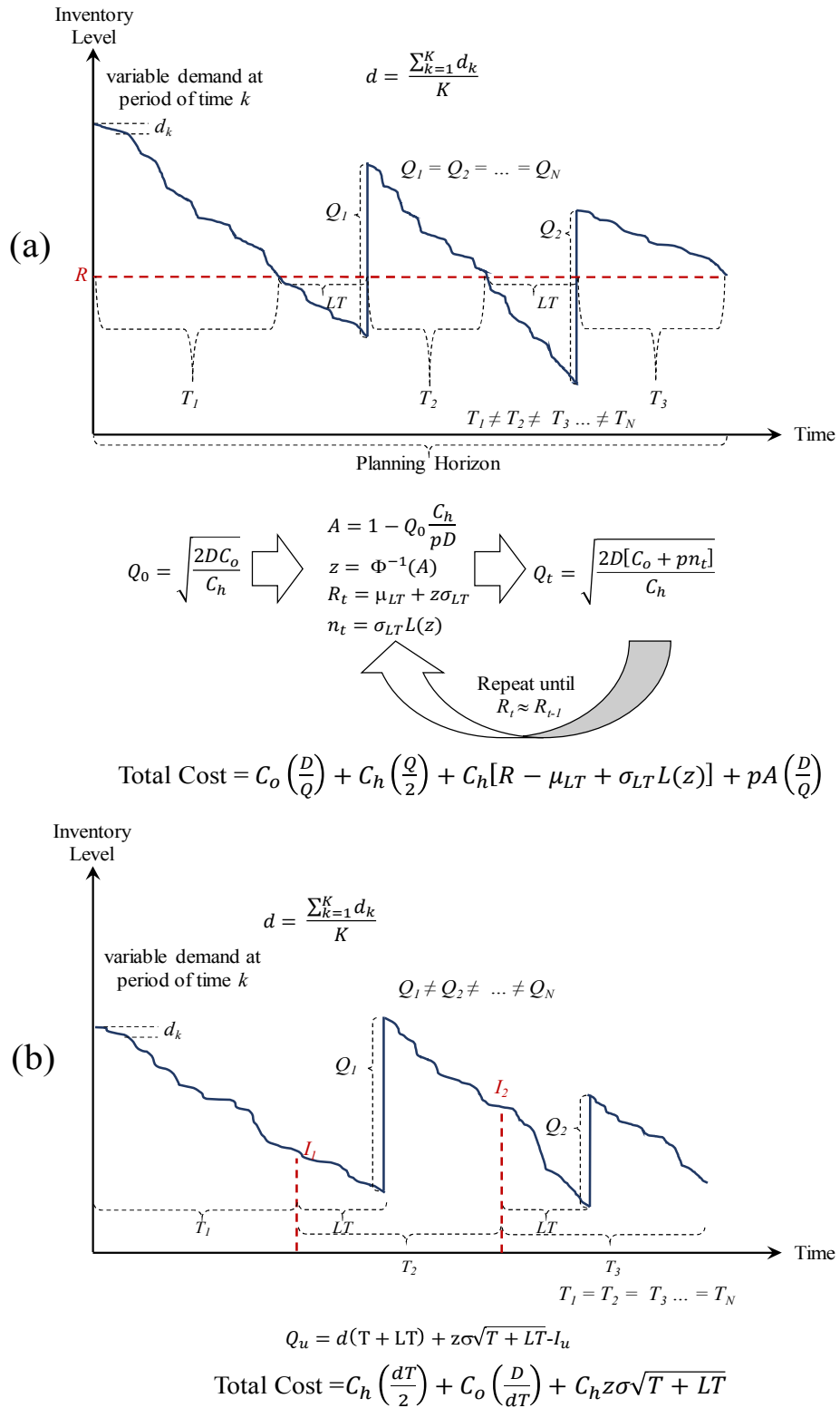


Fig. 1. Inventory planning strategies: (a) continuous review (Q, R) model, (b) periodic review (P) model

**Table 1. Main parameters and variables of the (Q, R) and P models**

Variable	Description
$D$	Cumulative demand through a planning horizon.
$C_o$	Order cost per lot.
$C_h$	Holding cost per unit in inventory.
$p$	Stock-out cost per unit.
$Q$	Economic order quantity (optimal lot size).
$R$	Reorder point (level of inventory to file an order to avoid stock-out).
$\mu_{LT}$	Average demand through the lead time.
$\sigma_{LT}$	Standard deviation through the lead time.
$LT$	Lead time
$T$	Time between inventory reviews and $T > LT$ .
$d$	Average daily demand, or average demand in the smallest unit of time ( $k \times d = D$ ) where $k$ is the number of days, weeks, of months within the planning horizon.
$\sigma$	Standard deviation of the average daily demand.
$L(z)$	Loss function associated to the re-order point.
$z$	Number of standard deviations associated to a service level.
$C$	Purchase cost per unit.

## 2.2 Non-deterministic Inventory Models

Two of the most useful techniques for inventory control are the continuous and periodic review models with non-deterministic demand. These models, also known as the (Q, R) [7,8] and P [6] models respectively, consider the costs and variables described in Table 1.

With these variables, lots of size  $Q$  (i.e., the EOQ) are ordered through a planning horizon to serve a cumulative demand. Fig. 1 presents an overview of the supply schemes considered by both models and their mathematical formulation which consists of expressions to determine  $Q$  and the total operational costs. Note that in the (Q, R) model, inventory review is performed continuously to detect the reorder point  $R$ . A constant lot of size  $Q$  is ordered when the inventory reaches  $R$ . As uncertain demand is assumed, the inventory consumption rate is different throughout the planning horizon, thus,  $R$  is reached at different times. This is the reason to obtain different times between inventory reviews. On the other hand, in the P model, inventory review is performed at fixed intervals and  $Q$  is estimated considering the available inventory  $I$  at that moment. Thus, different lots of size  $Q$  are ordered depending of the available inventory at the end of the review period  $T$ .

## 3. SIMULATION CODES AND RESULTS

Dynamic implementation of the inventory models was performed in the open source programming software GNU Octave [16]. The advantage of this

software is that it runs on GNU/Linux, macOS, BSD, and Windows, and it is compatible with many MATLAB scripts. Fig. 2 presents the code that estimates the output parameters  $Q$  and  $R$  for the (Q, R) model based on its mathematical formulation (see Fig. 1(a)). As input parameters the model considers:

- (i). The cost parameters:  $C_h$ ,  $C_o$ ,  $C$  and  $p$ ;
- (ii). The demand parameters:  $d$  (daily average demand),  $std$  (the standard deviation of the demand,  $\sigma$ ) and  $D$  (total cumulative demand which is equal to  $k \times d$  where  $k$  is the number of working days within the planning horizon);
- (iii). The time parameters:  $LT$  (lead time) and  $T$  (time between reviews).

While for the (Q, R) model the cost and time parameters are constant, demand is assumed with significant uncertainty. Thus, the average and standard deviation of the demand are considered as variables of interest. Also, the number of standard deviations of demand variability  $w$  (i.e.,  $z$ ) is considered as variable of interest due to its importance to reduce stock-out risks.

Once the output parameters,  $Q$  and  $R$ , are estimated, it is necessary to visualize how they will be able to ensure appropriate inventory levels to avoid stock-out in the presence of variable demand. For this purpose, the code presented in Fig. 3 was developed. Complementary comments are added to describe more accurately the objective of each programming line. In general, this code

generates variable demand data within the range  $\hat{d} = d \pm w\sigma$  which progressively reduces the initial inventory level (inventory consumption process). Once the reorder point  $R$  is reached, the order operation is set to re-supply the inventory with  $Q$  additional units at the end of the lead time  $LT$ . This variable consumption and replenishment pattern are plotted through the planning horizon.

Fig. 4(a) and Fig. 4(b) present the inventory levels for the  $(Q, R)$  model if demands are variable with the conditions  $w = \pm 3.0$  and  $w = \pm 5.0$  standard deviations for the example with the input data shown in Fig. 2. As observed, if variability increases, the inventory policy

determined with  $Q$  and  $R$  may lose the ability to avoid stock-out.

Then, Fig. 5 presents the code to estimate the output parameter  $Q$  for the Periodic Review  $(P)$  model. This code also generates variable demand data within the range  $\hat{d} = d \pm w\sigma$  which progressively reduces the initial inventory level (inventory consumption process). Once the period between reviews  $T$  is reached, the lot size  $Q$  is requested considering the available inventory  $I$  at that point. Then, at the end of the lead time  $LT$ , the inventory is re-supplied with  $Q/I$  additional units. This variable consumption and replenishment pattern are plotted through the planning horizon.

```

*Inventory_Models.m
1 clear all; clc; pkg load statistics
2 d = 5; std = 3; %units per day of average demand and standard deviation
3 w = 3; %standard deviations for demand variability
4 k = 270; D = k*d; %working days and cumulative demand
5 C= 230.0 ; Co = 250.0 ; Ch = 0.10*C ; p= 0.70*C; % Inventory associated costs
6 LT = 5; T = 15; % days of lead time and days between reviews
7 Z_p = norminv(0.95); % Z for Periodic Review Model
8 % ===== Determine parameters for (Q,R) model =====
9 for i=1:10 %Number of iterations
10 if i==1 % Initial Value for Q_qr
11 Q_qr = sqrt((2*D*Co)/Ch); else Q_qr = sqrt((2*D*(Co + p*n))/Ch);
12 end
13 A=1-Q_qr*(Ch/(p*D)); Z_qr=norminv(A); % Z for Continuous Review Model
14 R_qr = d*LT + Z_qr*std*sqrt(LT);
15 L_Z = stdnormal_pdf(Z_qr)-Z_qr*(1-stdnormal_cdf(Z_qr)); % Loss function
16 n = L_Z*std*sqrt(LT);
17 end
18 Q_qr=ceil(Q_qr); R_qr=ceil(R_qr);% round final values towards positive infinity
19
20
    
```

Fig. 2. Simulation model 1: Estimation of  $Q$  and  $R$  for the continuous review  $(Q, R)$  model

```

Inventory_Models.m
20
21 % ===== Test (Q,R) model =====
22 Inventory = Q_qr+R_qr; count_LT=0;
23 inv_consumption = [];
24 for i=1:k
25 inv_consumption = [inv_consumption ; Inventory];
26 dt=d+unifrnd(-w,w)*std; % Random daily demand within w std
27 if dt<0 dt = 0; end;
28 if Inventory - dt > R_qr
29 Inventory =Inventory - dt; else Inventory =Inventory - dt;
30 count_LT=count_LT+1;
31 end
32 if count_LT == LT+1
33 Inventory=Inventory+Q_qr; count_LT=0;
34 end
35 end
36 vect_R_qr=ones(length(inv_consumption),1)*R_qr;
37 plot(inv_consumption); hold on; plot(vect_R_qr, '-r'); axis([0 k 0 Q_qr+R_qr]);
38 xlabel('Working days'); ylabel('Inventory level');
39
    
```

Fig. 3. Simulation model 2: Simulation of inventory consumption and re-supply for the continuous review  $(Q, R)$  model

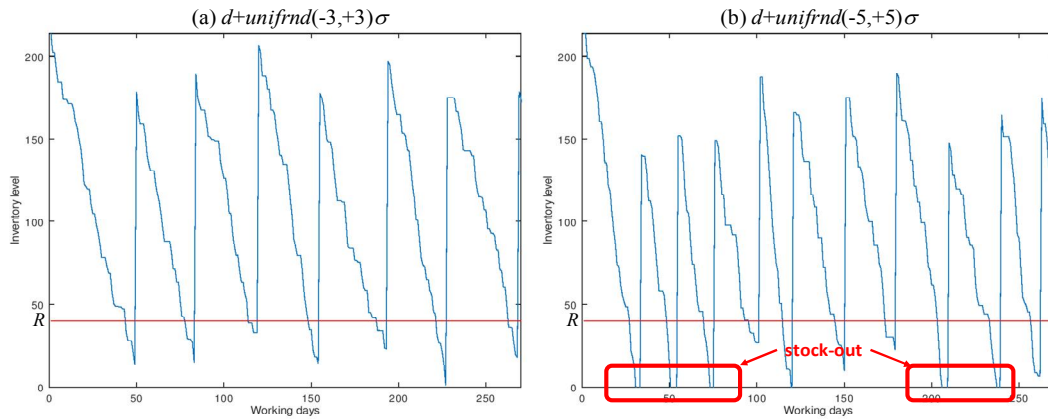


Fig. 4. Inventory and supply patterns with simulation model 1-2: (a)  $w = 3$ , (b)  $w = 5$

```

Inventory_Models.m
39
40 % ===== Determine parameters for P model and test =====
41 I = 0; Q_p=d*(T+LT)+Z_p*std*sqrt(T+LT)-I; Inventory=Q_p; count_T=0; count_LT=0;
42 inv_consumption = [];
43 for i=1:k
44     inv_consumption = [inv_consumption ; Inventory];
45     dt=d+unifrnd(-w,w)*std; % Random daily demand within w std
46     if dt<0 dt = 0; end;
47     Inventory=Inventory-dt; count_T=count_T+1;
48     if count_T == T; I = Inventory; count_T=0; count_LT=LT+1; end
49     if count_LT>0
50         count_LT=count_LT-1;
51         if count_LT == 0; Inventory = Inventory+(Q_p-I); end
52     end
53 end
54 figure
55 plot(inv_consumption); axis([0 k 0 Q_p]);
56 xlabel('Working days'); ylabel('Inventory level');
57
    
```

Fig. 5. Simulation model 3: Estimation of Q and simulation of inventory consumption and re-supply for the periodic review (P) model

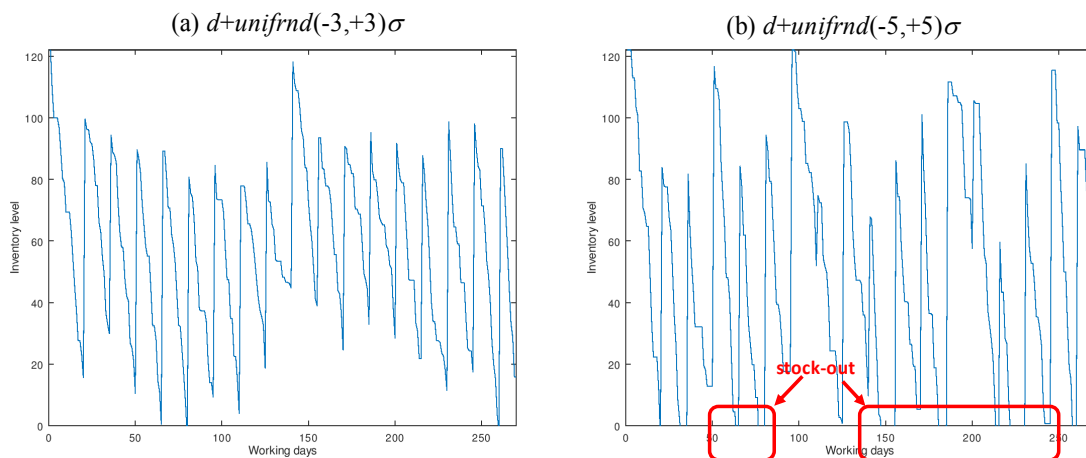


Fig. 6. Inventory and supply patterns with simulation model 3: (a)  $w = 3$ , (b)  $w = 5$

Fig. 6(a) and 6(b) present the inventory levels if demands are variable with the conditions  $w = \pm 3.0$  and  $w = \pm 5.0$  standard deviations and the same input data for the (Q, R) model is considered. As previously observed, if variability increases the inventory policy may lose the ability to avoid stock-out. Thus, these codes can support additional adjustment and / or evaluation of these inventory supply strategies.

#### 4. CONCLUSIONS

In this work three simulation codes were presented to dynamically evaluate the performance of two inventory control techniques for uncertain demand. (non-deterministic) demand. As presented in Figs. 4 and 6 the codes accurately identify the break point of the inventory supply strategy for large demand uncertainty and estimate the number of orders that may be placed through a planning horizon.

As these codes can be used to simulate the inventory consumption pattern with non-deterministic demand, they can be useful to extend on the understanding of these models to teach them, improve them, applying them to a specific business, and evaluate them when the conditions of the demand change. In such case, being able to estimate the performance of the model can lead to the appropriate changes in the supply strategy and reduce the stock-out risks.

As research source material, the present code can be used to develop integral free software for inventory procurement with distribution planning. This can lead to improve more aspects of the supply chain of micro, small and medium enterprises.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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